JINDAL ADARSH GRAMYA BHARTI

HR. SEC. SCHOOL, KIRODIMAL

NAGAR

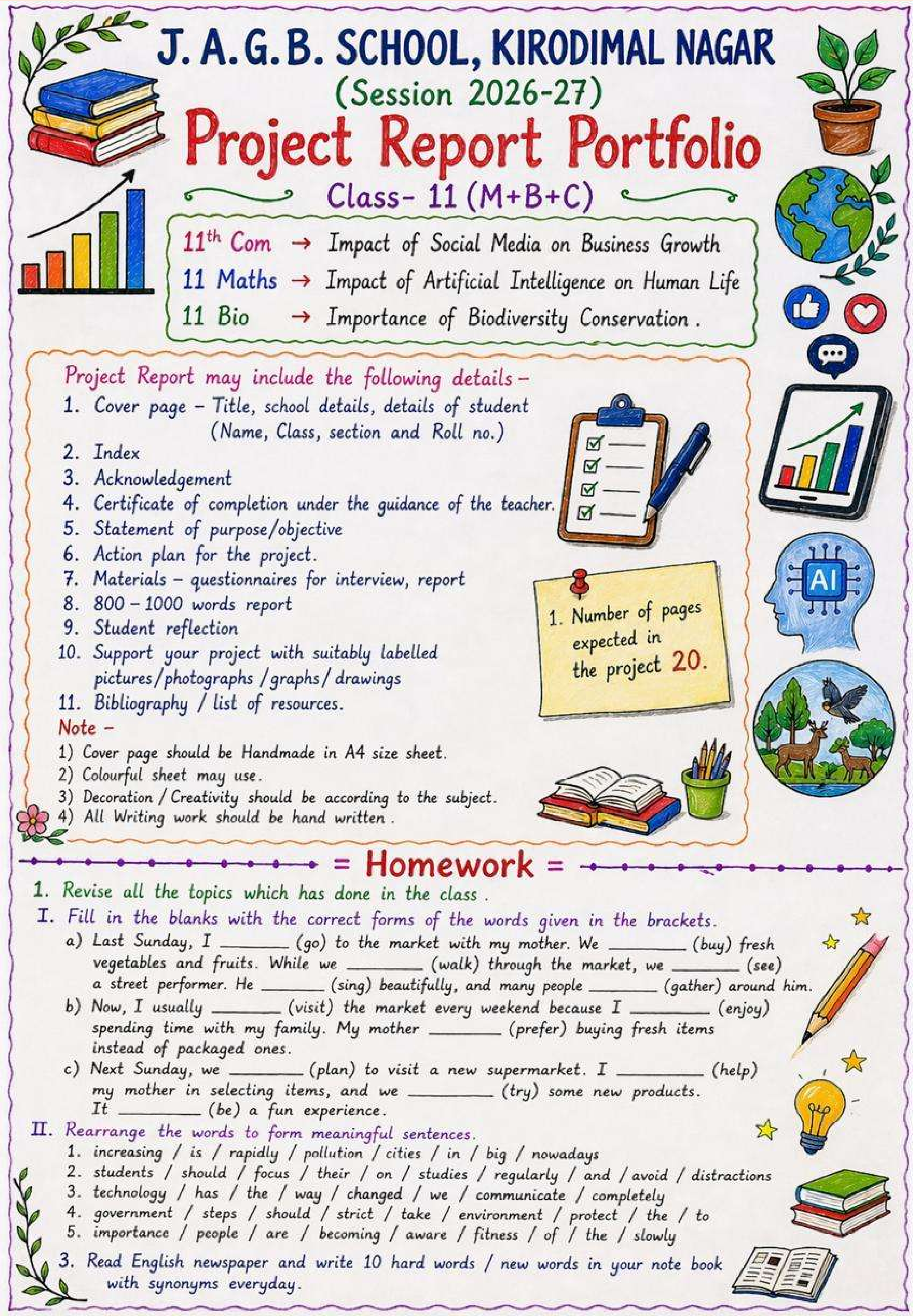
SUMMER VACATION HOMEWORK -

2026

CLASS - XI

CBSE

SUBJECT – ENGLISH



J. A. G. B. SCHOOL, KIRODIMAL NAGAR

(Session 2026-27)

Project Report Portfolio

Class- 11 (M+B+C)

11th Com → Impact of Social Media on Business Growth
11 Maths → Impact of Artificial Intelligence on Human Life
11 Bio → Importance of Biodiversity Conservation.

Project Report may include the following details –

1. Cover page – Title, school details, details of student (Name, Class, section and Roll no.)
2. Index
3. Acknowledgement
4. Certificate of completion under the guidance of the teacher.
5. Statement of purpose/objective
6. Action plan for the project.
7. Materials – questionnaires for interview, report
8. 800 – 1000 words report
9. Student reflection
10. Support your project with suitably labelled pictures/photographs/graphs/drawings
11. Bibliography / list of resources.

1. Number of pages expected in the project 20.

Note –

1) Cover page should be Handmade in A4 size sheet.
2) Colourful sheet may use.
3) Decoration / Creativity should be according to the subject.
4) All Writing work should be hand written.

= Homework =

1. Revise all the topics which has done in the class.

I. Fill in the blanks with the correct forms of the words given in the brackets.

a) Last Sunday, I ______ (go) to the market with my mother. We ______ (buy) fresh vegetables and fruits. While we ______ (walk) through the market, we ______ (see) a street performer. He ______ (sing) beautifully, and many people ______ (gather) around him.

b) Now, I usually ______ (visit) the market every weekend because I ______ (enjoy) spending time with my family. My mother ______ (prefer) buying fresh items instead of packaged ones.

c) Next Sunday, we ______ (plan) to visit a new supermarket. I ______ (help) my mother in selecting items, and we ______ (try) some new products. It ______ (be) a fun experience.

II. Rearrange the words to form meaningful sentences.

1. increasing / is / rapidly / pollution / cities / in / big / nowadays
2. students / should / focus / their / on / studies / regularly / and / avoid / distractions
3. technology / has / the / way / changed / we / communicate / completely
4. government / steps / should / strict / take / environment / protect / the / to
5. importance / people / are / becoming / aware / fitness / of / the / slowly

3. Read English newspaper and write 10 hard words / new words in your note book with synonyms everyday.

SUBJECT –MATHS

Summer Vacation Homework

Class 11 Maths

SECTION – A

Questions 1 to 10 carry 1 mark each.

1. The number of subsets of a set containing n elements is

(a) 2^n (b) $2^n - 1$ (c) $2^n - 1$ (d) n^n

2. Let A = {2, 5}, then subsets of set A are ϕ, {2}, {5}, {2, 5}, i.e. 4 subsets then the number of elements its power set contains are

(a) 4 (b) 42 (c) 10 (d) 2

3. The set $(A \cap B)' \cup (B \cap C)$ is equal to

(a) $A' \cup B \cup C$ (b) $A' \cup B$ (c) $A' \cup C'$ (d) $A' \cap B$

4. Let S = set of all points inside the square, T = the set of points inside the triangle and C = the set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then

(a) $S \cap T \cap C = \phi$ (b) $S \cup T \cup C = C$ (c) $S \cup T \cup C = S$ (d) $S \cup T = S \cap C$

5. If set A: numbers multiple of 4 and set B: numbers multiple of 6, then set $A \cap B$ is

(a) numbers multiple of 2 (b) numbers multiple of 4

(c) numbers multiple of 12 (d) numbers multiple of 24

6. For disjoint sets A and B, n(A) = 3, n(B) = 5, then $n(A \cap B)$ is

7. Representation of set $A = \{x \mid x \in Z, x^2 < 20\}$ in the roster form is

(a) {1, 2, 3, ..., 20} (b) {1, 2, 3, 4}

(c) {0, 1, 2, 3, 4} (d) {−4, −3, −2, −1, 0, 1, 2, 3, 4}

8. The set {−1, 1} in the set builder form can be written as

(a) {−1, 1} (b) $\{x \in W : x \leq 1\}$

(c) $\{x \in Z : x \leq 1\}$ (d) $\{x : x \text{ is a solution of } x^2 = 1\}$

For Q9 and Q10, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

9. **Assertion (A):** The set A = {x : x is an even prime number greater than 2} is the empty set.
Reason (R): The set B = {x : x^2 = 4, x is odd} is not an empty set.

10. **Assertion (A):** If n(A) = 3, n(B) = 6 and A ⊂ B, then the number of elements in A ∪ B is 9.
Reason (R): If A and B are disjoint, then n(A ∪ B) is n(A) + n(B) .

SECTION – B

Questions 11 to 14 carry 2 marks each.

11. A and B are two sets such that : n(A – B) = 14 +x, n(B – A) = 3x and n(A ∩ B) = x. draw a Venn diagram to illustrate information and if n(A) = n(B) then find the value of x.

12. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

13. A and B are two sets such that n(A) = 3 and n(B) = 6. Find (*i*) minimum value of n(A ∪ B) (*ii*) maximum value of n(A ∪ B).

14. If U = {x : $x \leq$ 10, $x \in$ N}, A = {x : $x \in$ N, x is prime}, B = {x : $x \in$ N, x is even}, write A ∩ B' in roster form.

SECTION – C

Questions 15 to 17 carry 3 marks each.

15. In an examination, 80% students passed in Mathematics, 72% passed in Science and 13% failed in both the subjects, if 312 students passed in both the subjects. Find the total number of students who appeared in the examination.

16. Let A, B and C be three sets such that A ∪ B = A ∪ C and A ∩ B = A ∩ C. Show that B = C.

17. Let U = {1, 2, 3, 4, 5, 6, 8}, A = {2, 3, 4}, B = {3, 4, 5}. Show that (A ∪ B)' = A' ∩ B' and (A ∩ B)' = A' ∪ B'

SECTION – D

Questions 18 carry 5 marks.

18. In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows :
French = 17, English = 13, Sanskrit = 15
French and English = 9, English and Sanskrit = 4
French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study
(i) French only
(ii) English only
(iii) Sanskrit only
(iv) English and Sanskrit but not French
(v) French and Sanskrit but not English

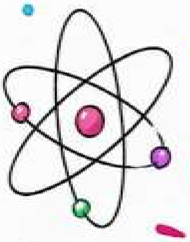
SECTION – E (Case Study Based Questions)

Questions 19 to 20 carry 4 marks each.

19. In a city of 56,000 people, following is the number of fans of players Rohit (R), Virat (V) and Dhoni (D):

Players	Number of Fans
Rohit	23,000
Virat	25,000
Dhoni	18,000
Rohit and Virat	12,000
Rohit and Dhoni	10,000
Virat and Dhoni	8,000
Rohit, Virat and Dhoni	3,000

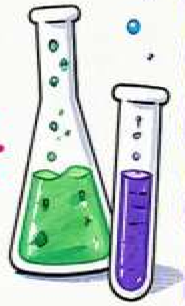
INSTRUCTIONS: **Practice these questions on their respective Homework copy.**



Class 11

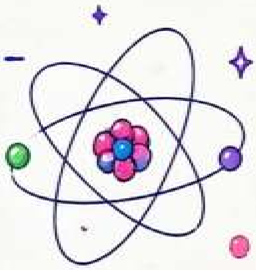
Holiday Homework

Chemistry



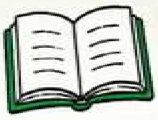
1. Write the NCERT solutions of **Chapter 1** in your chemistry notebook.

2. Make a proper notes of the chapter – **Structure of Atoms.**

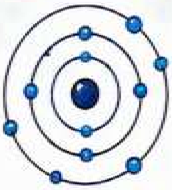


Your notes should include:

Headings & Subheadings



Diagrams



Formulas

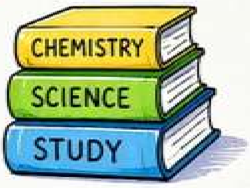
$E = h\nu$

$\lambda = \frac{h}{mv}$

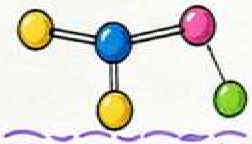
Definitions

3. Ensure that your notes are **spacious** and **neatly presented**, with additional relevant points included wherever necessary.

4. Read all the assigned chapters **thoroughly** for better understanding.



Stay Curious,
Keep Learning!



1.6 Calculate the concentration of nitric acid in moles per litre in a sample which has a density, 1.41 g mL^{-1} and the mass per cent of nitric acid in it being 69%.

1.7 How much copper can be obtained from 100 g of copper sulphate ($CuSO_4$)?

1.8 Determine the molecular formula of an oxide of iron, in which the mass per cent of iron and oxygen are 69.9 and 30.1, respectively.

1.9 Calculate the atomic mass (average) of chlorine using the following data:

	% Natural Abundance	**Molar Mass**
^{35}Cl	75.77	34.9689
^{37}Cl	24.23	36.9659

1.10 In three moles of ethane (C_2H_6), calculate the following:

(i) Number of moles of carbon atoms.

(ii) Number of moles of hydrogen atoms.

(iii) Number of molecules of ethane.

1.11 What is the concentration of sugar ($C_{12}H_{22}O_{11}$) in mol L^{-1} if its 20 g are dissolved in enough water to make a final volume up to 2L?

1.12 If the density of methanol is 0.793 kg L^{-1}, what is its volume needed for making 2.5 L of its 0.25 M solution?

1.13 Pressure is determined as force per unit area of the surface. The SI unit of pressure, pascal is as shown below:

$1\text{Pa} = 1\text{N m}^{-2}$

If mass of air at sea level is 1034 g cm^{-2}, calculate the pressure in pascal.

1.14 What is the SI unit of mass? How is it defined?

1.15 Match the following prefixes with their multiples:

	Prefixes	**Multiples**
(i)	micro	10^6
(ii)	deca	10^9
(iii)	mega	10^{-6}
(iv)	giga	10^{-15}
(v)	femto	10

1.16 What do you mean by significant figures?

1.17 A sample of drinking water was found to be severely contaminated with chloroform, $CHCl_3$, supposed to be carcinogenic in nature. The level of contamination was 15 ppm (by mass).

(i) Express this in per cent by mass.

(ii) Determine the molality of chloroform in the water sample.

1.18 Express the following in the scientific notation:

(i) 0.0048

(ii) 234,000

(iii) 8008

(iv) 500.0

(v) 6.0012

1.19 How many significant figures are present in the following?

(i) 0.0025

(ii) 208

(iii) 5005

(iv) 126,000

(v) 500.0

(vi) 2.0034

1.20 Round up the following upto three significant figures:

(i) 34.216

(ii) 10.4107

(iii) 0.04597

(iv) 2808

1.21 The following data are obtained when dinitrogen and dioxygen react together to form different compounds:

	Mass of dinitrogen	**Mass of dioxygen**
(i)	14 g	16 g
(ii)	14 g	32 g
(iii)	28 g	32 g
(iv)	28 g	80 g

(a) Which law of chemical combination is obeyed by the above experimental data? Give its statement.

(b) Fill in the blanks in the following conversions:

(i) 1 km = ... mm = ... pm

(ii) 1 mg = ... kg = ... ng

(iii) 1 mL = ... L = ... dm^3

1.22 If the speed of light is 3.0×10^8 m s^{-1}, calculate the distance covered by light in 2.00 ns.

1.23 In a reaction

$A + B_2 \rightarrow AB_2$

Identify the limiting reagent, if any, in the following reaction mixtures.

(i) 300 atoms of A + 200 molecules of B

(ii) 2 mol A + 3 mol B

(iii) 100 atoms of A + 100 molecules of B

(iv) 5 mol A + 2.5 mol B

(v) 2.5 mol A + 5 mol B

1.24 Dinitrogen and dihydrogen react with each other to produce ammonia according to the following chemical equation:

$N_2 (g) + H_2 (g) \rightarrow 2NH_3 (g)$

(i) Calculate the mass of ammonia produced if 2.00×10^3 g dinitrogen reacts with 1.00×10^3 g of dihydrogen.

(ii) Will any of the two reactants remain unreacted?

(iii) If yes, which one and what would be its mass?

1.25 How are 0.50 mol Na_2CO_3 and 0.50 M Na_2CO_3 different?

1.26 If 10 volumes of dihydrogen gas reacts with five volumes of dioxygen gas, how many volumes of water vapour would be produced?

1.27 Convert the following into basic units:

(i) 28.7 pm

(ii) 15.15 pm

(iii) 25365 mg

1.28 Which one of the following will have the largest number of atoms?

(i) 1 g Au (s)

(ii) 1 g Na (s)

(iii) 1 g Li (s)

(iv) 1 g of Cl_2(g)

1.29 Calculate the molarity of a solution of ethanol in water, in which the mole fraction of ethanol is 0.040 (assume the density of water to be one).

1.30 What will be the mass of one ^{12}C atom in g?

1.31 How many significant figures should be present in the answer of the following calculations?

(i) $\frac{0.02856 \times 298.15 \times 0.112}{0.5785}$ (ii) 5×5.364

(iii) $0.0125 + 0.7864 + 0.0215$

1.32 Use the data given in the following table to calculate the molar mass of naturally occuring *argon* isotopes:

Isotope	**Isotopic molar mass**	**Abundance**
^{36}Ar	35.96755 g mol^{-1}	0.337%
^{38}Ar	37.96272 g mol^{-1}	0.063%
^{40}Ar	39.9624 g mol^{-1}	99.600%

1.33 Calculate the number of atoms in each of the following (i) 52 moles of Ar (ii) 52 u of He (iii) 52 g of He.

1.34 A welding fuel gas contains carbon and hydrogen only. Burning a small sample of it in oxygen gives 3.38 g carbon dioxide, 0.690 g of water and no other products. A volume of 10.0 L (measured at STP) of this welding gas is found to weigh 11.6 g. Calculate (i) empirical formula, (ii) molar mass of the gas, and (iii) molecular formula.

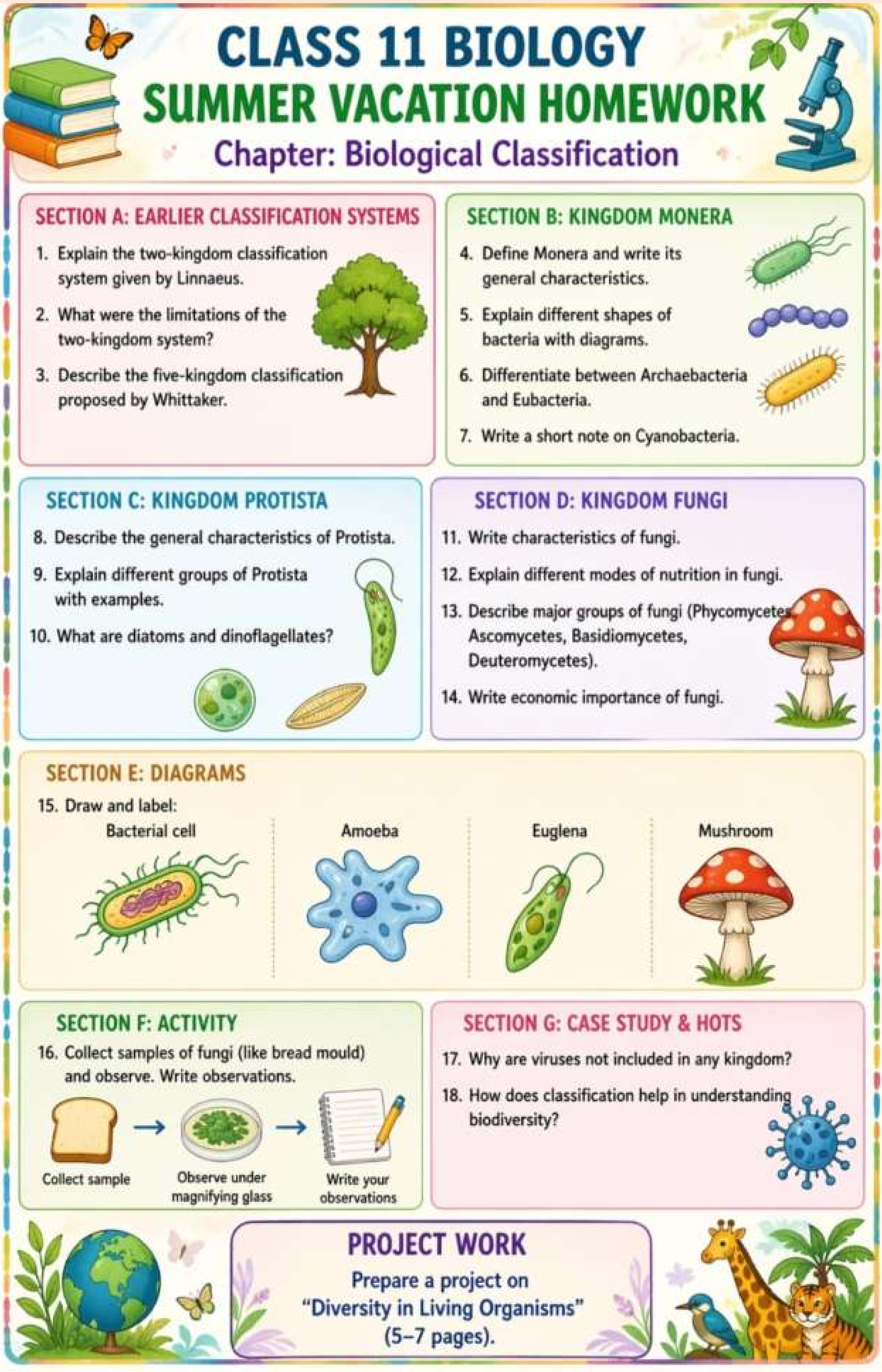
1.35 Calcium carbonate reacts with aqueous HCl to give $CaCl_2$ and CO_2 according to the reaction, $CaCO_3$ (s) + 2 HCl (aq) → $CaCl_2$ (aq) + CO_2(g) + H_2O(l)

What mass of $CaCO_3$ is required to react completely with 25 mL of 0.75 M HCl?

1.36 Chlorine is prepared in the laboratory by treating manganese dioxide (MnO_2) with aqueous hydrochloric acid according to the reaction

4 HCl (aq) + MnO_2(s) → $2H_2O$ (l) + $MnCl_2$(aq) + Cl_2 (g)

How many grams of HCl react with 5.0 g of manganese dioxide?



CLASS 11 BIOLOGY
SUMMER VACATION HOMEWORK
Chapter: Biological Classification

SECTION A: EARLIER CLASSIFICATION SYSTEMS

1. Explain the two-kingdom classification system given by Linnaeus.
2. What were the limitations of the two-kingdom system?
3. Describe the five-kingdom classification proposed by Whittaker.

SECTION B: KINGDOM MONERA

4. Define Monera and write its general characteristics.
5. Explain different shapes of bacteria with diagrams.
6. Differentiate between Archaebacteria and Eubacteria.
7. Write a short note on Cyanobacteria.

SECTION C: KINGDOM PROTISTA

8. Describe the general characteristics of Protista.
9. Explain different groups of Protista with examples.
10. What are diatoms and dinoflagellates?

SECTION D: KINGDOM FUNGI

11. Write characteristics of fungi.
12. Explain different modes of nutrition in fungi.
13. Describe major groups of fungi (Phycomycetes, Ascomycetes, Basidiomycetes, Deuteromycetes).
14. Write economic importance of fungi.

SECTION E: DIAGRAMS

15. Draw and label:

Bacterial cell | Amoeba | Euglena | Mushroom

SECTION F: ACTIVITY

16. Collect samples of fungi (like bread mould) and observe. Write observations.

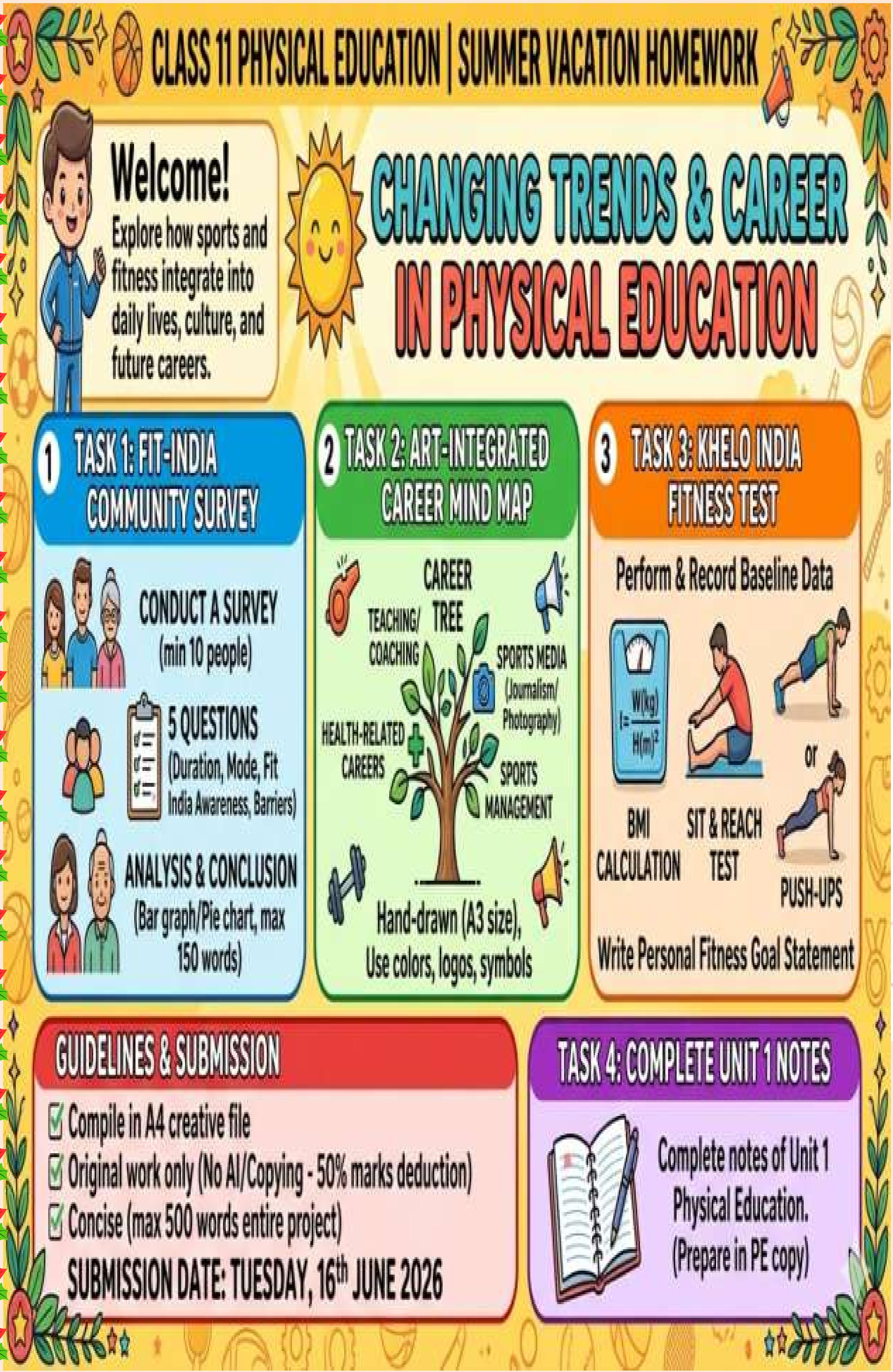
Collect sample → Observe under magnifying glass → Write your observations

SECTION G: CASE STUDY & HOTS

17. Why are viruses not included in any kingdom?
18. How does classification help in understanding biodiversity?

PROJECT WORK

Prepare a project on
"Diversity in Living Organisms"
(5–7 pages).



CLASS 11 PHYSICAL EDUCATION | SUMMER VACATION HOMEWORK

Welcome!

Explore how sports and fitness integrate into daily lives, culture, and future careers.

CHANGING TRENDS & CAREER IN PHYSICAL EDUCATION

1 TASK 1: FIT-INDIA COMMUNITY SURVEY

CONDUCT A SURVEY (min 10 people)

5 QUESTIONS (Duration, Mode, Fit India Awareness, Barriers)

ANALYSIS & CONCLUSION (Bar graph/Pie chart, max 150 words)

2 TASK 2: ART-INTEGRATED CAREER MIND MAP

CAREER TREE

TEACHING/ COACHING

SPORTS MEDIA (Journalism/ Photography)

HEALTH-RELATED CAREERS

SPORTS MANAGEMENT

Hand-drawn (A3 size), Use colors, logos, symbols

3 TASK 3: KHELO INDIA FITNESS TEST

Perform & Record Baseline Data

$I = \frac{W(kg)}{H(m)^2}$

BMI CALCULATION

SIT & REACH TEST

PUSH-UPS or

Write Personal Fitness Goal Statement

GUIDELINES & SUBMISSION

- ☑ Compile in A4 creative file
- ☑ Original work only (No AI/Copying - 50% marks deduction)
- ☑ Concise (max 500 words entire project)

SUBMISSION DATE: TUESDAY, 16th JUNE 2026

TASK 4: COMPLETE UNIT 1 NOTES

Complete notes of Unit 1 Physical Education. (Prepare in PE copy)

★ INFORMATICS PRACTICES ASSIGNMENT ★

CLASS 11th

Write Python programs for the following questions.
Show proper logic and output clearly.

1 Write a program to print "Hello, World!" on the screen.

Logic: Use the print() function to display the text exactly as shown.

2 Write a program to take your name as input and print a greeting message.

Logic: Use input() to take the name and print a personalized greeting.

3 Write a program to add two numbers entered by the user.

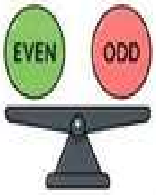
Logic: Take two inputs, convert them to numbers, add them and print the result.

4 Write a program to find the square of a number.

Logic: Take a number as input, square it using * operator and print the result.

5 Write a program to check whether a number is even or odd.

Logic: Use modulus operator (%) to check and print whether the number is even or odd.



6 Write a program to find the largest of two numbers.

Logic: Take two numbers as input, compare them using if-else and print the largest number.

★ DO YOUR BEST – CODING IS FUN WHEN YOU TRY YOUR BEST! ★

Write Python programs for the following questions.
Show proper logic and output clearly.

7 Write a program to find the largest of two numbers.

Logic: Take two numbers as input → use if-else to compare → print the larger number.

8 Write a program to check whether a number is positive, negative, or zero.

Logic: Take a number → use if-elif-else → check >0, <0, or =0 → print result.

9 Write a program to check whether a number is divisible by 5.

Logic: Use modulus operator (%) → if number % 5 == 0 → divisible, else not divisible.

10 Write a program to print your favorite quote on the screen.

Logic: Use print() function to display the quote.

11 Write a program to take your age as input and display it.

Logic: Use input() to take age → print the entered value.

12 Write a program to subtract two numbers entered by the user.

Logic: Take two inputs → convert to numbers → subtract → print result.

13 Write a program to multiply two numbers entered by the user.

Logic: Take two inputs → convert to numbers → multiply them → print the result.

14 Write a program to divide two numbers and display the result.

Logic: Take two inputs → divide first by second → print result (check division by zero).

15 Write a program to find the cube of a number.

Logic: Take a number → multiply it by itself three times (n * n * n) → print result.

Mathematical Tools

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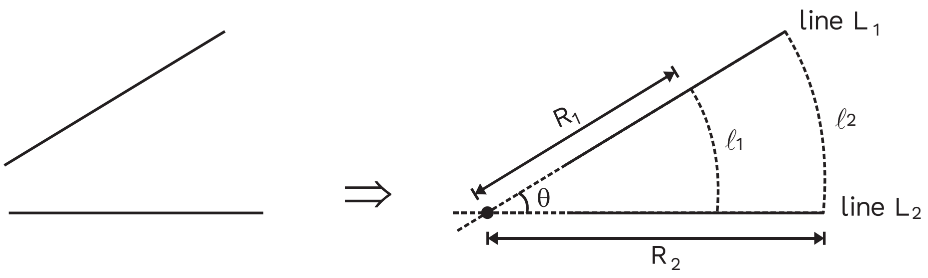
Trigonometry

Physics is basically a combination of theory and calcuations.
For “calculations”, we need to learn some mathematical tools.

Angle :



θ = angular gap or angle between line L_1 and L_2.
By definition,

$$\boxed{\text{angle } (\theta) = \frac{\text{Arc}(\ell)}{\text{Radius (R)}}}$$

In above diagram

$$\theta = \frac{\ell_1}{R_1} = \frac{\ell_2}{R_2}$$

The unit of angle is ‘radian’ (rad.)

Angle has two directions → Anti-clockwise (positive)
→ Clockwise (Negative)

If we rotate line in anticlockwise direction

→ From OA to OC (semi-circle),

$$\theta_1 = \frac{\text{length ABC}}{\text{length OA}} = 3.141....\text{rad} = \pi\,\text{rad} \approx \frac{22}{7}\text{rad}.$$

→ From OA to OB (quarter-circle),

$$\theta_2 = \frac{\text{arc length AB}}{\text{radius length OA}} = 1.57....\text{rad} = \frac{\pi}{2}\text{rad}.$$

Similarly, for complete circle,

$$\text{angle} = \frac{\text{circumference of circle}}{\text{Radius}}$$

$$\theta_{\text{circle}} = 6.282....\text{rad} = 2\pi\ \text{rad}.$$

- Angle can also be measured in 'degree' where

$$\pi \text{ rad} = 180° \quad \Rightarrow \quad 1 \text{ rad} = \frac{180°}{\pi} \approx 57.3°$$

$$\text{or} \quad 180° = \pi \text{ rad} \quad \Rightarrow \quad 1° = \left(\frac{\pi}{180}\right)\text{rad}$$

For example,

$$\frac{\pi}{6}\text{rad} = \frac{180°}{6} = 30° \quad \text{and} \quad \frac{\pi}{3}\text{rad} = \frac{180°}{3} = 60°$$

Trigonometric Ratio (T-Ratio) :

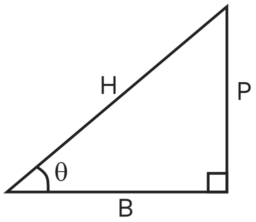
For a right-angled triangle,
Perpendicular (P) is that line which is just opposite to angle (θ), Hypotenuse (H) is that line which is just opposite to 90° angle while other line is base (B).

$$P^2 + B^2 = H^2$$

$$\sin\theta = \frac{P}{H}; \ \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{H}{P}$$

$$\cos\theta = \frac{B}{H}; \quad \sec\theta = \frac{1}{\cos\theta} = \frac{H}{B}$$

$$\tan\theta = \frac{P}{B}; \quad \cot\theta = \frac{1}{\tan\theta} = \frac{B}{P}$$



B → Base
P → Perpendicular
H → Hypotenuse

Also,

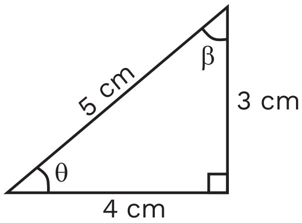
$$\tan\theta = \frac{P}{B} = \frac{P/H}{B/H} \quad \Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

On measuring, the angle θ = 37° and β = 53°
For given triangle,

$$\tan\theta = \frac{3}{4} \text{ while } \tan\beta = \frac{4}{3}$$

$$\sin\theta = \frac{3}{5} \text{ while } \sin\beta = \frac{4}{5}$$

$$\cos\theta = \frac{4}{5} \text{ while } \cos\beta = \frac{3}{5}$$



Trigonometric Identities :

$$\frac{P^2}{H^2} + \frac{B^2}{H^2} = \frac{H^2}{H^2} \quad \Rightarrow \quad \boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\frac{P^2}{B^2} + \frac{B^2}{B^2} = \frac{H^2}{B^2} \quad \Rightarrow \quad \boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\frac{P^2}{P^2} + \frac{B^2}{P^2} = \frac{H^2}{P^2} \quad \Rightarrow \quad \boxed{1 + \cot^2\theta = \mathrm{cosec}^2\theta}$$

Commonly used angles and their trigonometric ratios :

Angle (θ)	0°	30°	37°	45°	53°	60°	90°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{4}{5}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{4}{5}$	$\frac{1}{\sqrt{2}}$	$\frac{3}{5}$	$\frac{1}{2}$	0	–1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	$\frac{3}{4}$	1	$\frac{4}{3}$	$\sqrt{3}$	∞	0

Trigonometrical formulae :

We remember values of $\sin\theta$, $\cos\theta$ and $\tan\theta$, for
θ = {0°, 30°, 37°, 45°, 53°, 60°, 90°, 180°}

For example : $\sin 30^\circ = \frac{1}{2}$

But what if someone asks to calculate value of sin 120°.

→ Then we split the given angle in terms of
{0°, 30°, 37°, 45°, 53°, 60°, 90°, 180°}

For example : sin(120°) = sin(90° + 30°) = sin(180° – 60°)

- $\sin(2n\pi + \theta) = \sin\theta$, where n = 0, 1, 2, 3, ...

For example:

sin390° $= \sin(2\pi + 30^\circ)$ [∵ 2π = 360°]
$= \sin(2n\pi + 30^\circ)$ [Here, n = 1]
= sin 30°

$\sin 390^\circ = \frac{1}{2}$ $\left[\because \sin 30^\circ = \frac{1}{2}\right]$

Similarly,

- $\cos(2n\pi + \theta) = \cos\theta$
- $\tan(2n\pi + \theta) = \tan\theta$
- $\sin(\pi - \theta) = +\sin\theta$
- $\cos(\pi - \theta) = -\cos\theta$
- $\tan(\pi - \theta) = -\tan\theta$

Q. **Find sin120°**

Sol. $\sin 120° = \sin(180° - 60°) = \sin(\pi - 60°) = +\sin 60° = \frac{\sqrt{3}}{2}$

Q. **Find tan 150°**

Sol. $\tan 150° = \tan(180° - 30°) = \tan(\pi - 30°) = -\tan 30° = -\frac{1}{\sqrt{3}}$

$\sin(\pi + \theta) = -\sin\theta$
$\cos(\pi + \theta) = -\cos\theta$
$\tan(\pi + \theta) = +\tan\theta$

Q. **Find sin210°**

Sol. $\sin 210° = \sin(180° + 30°) = \sin(\pi + 30°) = -\sin 30° = -\frac{1}{2}$

$\sin(2\pi - \theta) = -\sin\theta$
$\cos(2\pi - \theta) = +\cos\theta$
$\tan(2\pi - \theta) = -\tan\theta$

Q. **Find sin330°**

Sol. sin330° = sin(360° – 30°) = $\sin(2\pi - 30°)$ = –sin 30° = $-\frac{1}{2}$

$$\sin\left(\frac{\pi}{2}+\theta\right) = +\cos\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

$$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$

Q. **Find tan 120°**

Sol. sin(120°) = sin(90° + 30°) $= \sin\left(\frac{\pi}{2}+30°\right)$ = +cos 30° = $\frac{\sqrt{3}}{2}$

$$\sin\left(\frac{\pi}{2}-\theta\right) = +\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = +\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = +\cot\theta$$

Q. **Find cos30° using above rule**

Sol. cos 30° = cos(90° – 60°) $= \cos\left(\frac{\pi}{2}-60°\right)$ = +sin 60° = $\frac{\sqrt{3}}{2}$

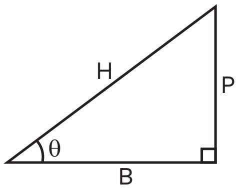
sin(–θ) = –sinθ
cos(–θ) = +cosθ
tan(–θ) = –tanθ

Q. **Find sin (–60°)**

Sol. sin(–60°) = –sin60° = $-\frac{\sqrt{3}}{2}$

Maximum and minimum value

In this right-angle triangle $P \le H$ and $B \le H$



$\sin\theta = \frac{P}{H}$

$\because \quad P \le H \quad \Rightarrow \frac{P}{H} \le 1$

$\therefore \quad \sin\theta \le 1$

Similarly, for negative angle

$\sin\theta \ge -1$

Combining above two inequalities

$-1 \le \sin\theta \le 1$

Now, $\cos\theta = \frac{B}{H}$

$\Rightarrow \quad \cos\theta \le 1$

Considering both negative and positive angles

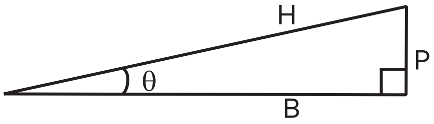
$-1 \le \cos\theta \le 1$

$\tan\theta = \frac{P}{B}$

$-\infty < \tan\theta < \infty$

Small angle approximation :

In a right-angle triangle, if θ is very small
$P << H$ and $H \simeq B$ (see the diagram)
$\therefore$ We consider P as arc and H and B as radius



$\rightarrow \quad \sin\theta = \frac{P}{H}$

$\sin\theta = \frac{\text{Arc}}{\text{Radius}}$

But we know that θ(in radians) $= \frac{\text{Arc}}{\text{Radius}}$

$\therefore \quad \boxed{\sin\theta = \theta, \text{ if } \theta \to 0} \quad \Rightarrow$ ‘θ’ should be in radian

Example :

sin 1° = 1 ✗ (incorrect) First convert 1° into radian $1^\circ = \frac{\pi}{180}\text{Rad}$ $\sin 1^\circ = \sin\left(\frac{\pi}{180}\right) = \frac{\pi}{180}$	→ $\cos\theta = \frac{B}{H}$ If θ is very small then, $H \simeq B$ $\Rightarrow \frac{B}{H} \simeq 1$ ∴ $\boxed{\cos\theta = 1,\ \text{if}\ \theta \to 0}$	→ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ For very small 'θ' sinθ = θ and cosθ = 1 ∴ For small 'θ' $\tan\theta = \frac{\theta}{1} = \theta$ $\boxed{\tan\theta = \theta,\ \text{if}\ \theta \to 0}$

Q. **Find value of sin 5°**

Sol. First convert 5° into radian.

$$1^\circ = \frac{\pi}{180}\ \text{Rad}$$

$$5^\circ = 5 \times \frac{\pi}{180}\ \text{Rad} \ = \frac{\pi}{36}\ \text{Rad}$$

$\frac{\pi}{36}$ Rad is a small angle

$$\therefore \quad \sin 5^\circ = \sin\left(\frac{\pi}{36}\right)$$

$$\sin 5^\circ = \frac{\pi}{36}$$

- sin(A + B) = sinA cosB + cosA sinB
  sin(A – B) = sinA cosB – cosA sinB
- cos(A + B) = cosA cosB – sinA sinB
  cos(A – B) = cosA cosB + sinA sinB

Q. **Find sin15° using above rule**

Sol. sin15° = sin(45° – 30°) = sin45° cos30° – cos45° sin30°

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\boxed{\sin 15^\circ = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

Q. **Find sin75° using above rule**

Sol. sin75° = sin(45° + 30°) = sin30° cos45° + cos30° sin45°

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\boxed{\sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}}$$

Q. **Find sin120° using above rule**

Sol. sin120° = sin(60° + 60°)

= sin60° cos60° + cos60° sin60°

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Q. **Find cos120° using above rule**

Sol. cos120° = cos(60° + 60°)

$$= \cos 60^\circ \cos 60^\circ - \sin 60^\circ \sin 60^\circ$$

$$= \frac{1}{2}\cdot\frac{1}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

$$\tan 120^\circ = \frac{\sin 120^\circ}{\cos 120^\circ}$$

Q. **Find cos106° using above rule**

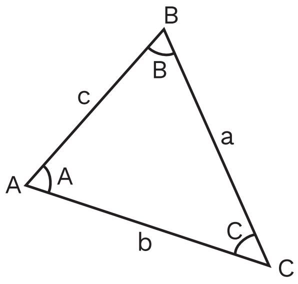
Sol. cos106° = cos(53° + 53°)

$$= \cos 53^\circ \cos 53^\circ - \sin 53^\circ \sin 53^\circ$$

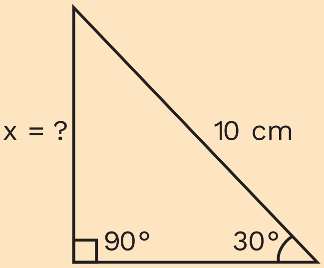
$$= \frac{3}{5}\cdot\frac{3}{5} - \frac{4}{5}\cdot\frac{4}{5} = \frac{-7}{25}$$

Sine formula :

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



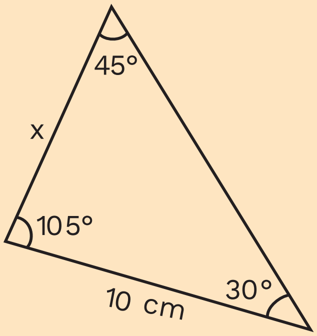
Q. **Find x in adjoining figure**



Sol. $\frac{\sin 90^\circ}{10} = \frac{\sin 30^\circ}{x}$

$\Rightarrow \quad x = 10 \times \frac{1}{2} = 5$

Q. **Find x in given figure**



Sol. $\frac{\sin 45^\circ}{10} = \frac{\sin 30^\circ}{x}$

$\Rightarrow \quad x = \frac{10 \times \sin 30^\circ}{\sin 45^\circ}$

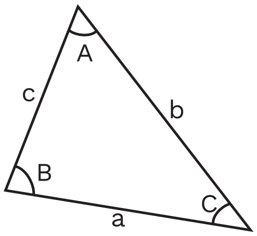
$= \frac{10 \times 1/2}{1/\sqrt{2}} = 5\sqrt{2}$

Cosine formula :

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Q. **Find θ in given figure**

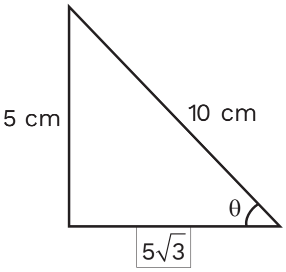
Sol.

$$\cos\theta = \frac{100 + 75 - 25}{2 \times 10 \times 5\sqrt{3}}$$

$$= \frac{150}{20 \times 5\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$



$$-\sqrt{a^2 + b^2} \le a\sin\theta + b\cos\theta \le +\sqrt{a^2 + b^2}$$

For Example :

y = 2 sinθ + 3 cosθ

Here a = 2, b = 3

$$y_{max} = \sqrt{2^2 + 3^2} = 13$$

$$y_{min} = -\sqrt{13}$$

$$-\sqrt{13} \le 2\sin\theta + 3\cos\theta \le \sqrt{13}$$

Q. **y = 3 sinθ + 4 cosθ**
Find maximum value of y.

Sol.

$$y_{max} = \sqrt{3^2 + 4^2}$$

= 5

Quadratic Equations

An equation of the form,

$$ax^2 + bx + c = 0, \quad a \neq 0 \text{ where}$$

a, b and c are constants and x is variable is called a quadratic equation.

Number of Solutions = Maximum Power of ‘x’

Solutions of equation are values of ‘x’ which when put in L.H.S, the L.H.S. will become zero.

Here, we have 2 solutions

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_1 + x_2 = -\frac{b}{a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \qquad x_1 \cdot x_2 = +\frac{c}{a}$$

For example :

In equation,

$$2x^2 + 3x - 6 = 0$$

$$a = 2, b = 3, c = -6$$

$$x_1 = \frac{-3 + \sqrt{3^2 - 4 \times 2 \times (-6)}}{2 \times 2} = \frac{-3 + \sqrt{57}}{4}$$

$$x_2 = \frac{-3 - \sqrt{3^2 - 4 \times 2 \times (-6)}}{2 \times 2} = \frac{-3 - \sqrt{57}}{4}$$

$$x_1 + x_2 = \frac{-3}{2}$$

$$x_1 \cdot x_2 = \frac{-6}{2} = -3$$

1) If $b^2 - 4ac > 0$

   Two real and distinct solutions or roots

2) If $b^2 - 4ac < 0$.

   Two imaginary and distinct solutions or roots

3) If $b^2 - 4ac = 0$

   Real and equal roots

- $\sqrt{-4} = \sqrt{4 \times (-1)} = 2 \times \sqrt{-1} = 2i$, Where $i^2 = -1$
- $\underbrace{x^2 + 2x - 8}_{\text{Expression}} = 0$

Same equation can be represent as $\underbrace{x^2 + 2x - 10}_{\text{Expression}} = -2$, $\underbrace{x^2 + 2x}_{\text{Expression}} = 8$

- When expression is equated with something it becomes equation.

  $x^2 + 2x - 8 = 0 \rightarrow$ Equation [Expression equated with 0]

  $x^2 + 2x = 8 \rightarrow$ Equation [Expression equated with 8]

- $y = x^2 \Rightarrow$ Equation of Parabola

When y = 4

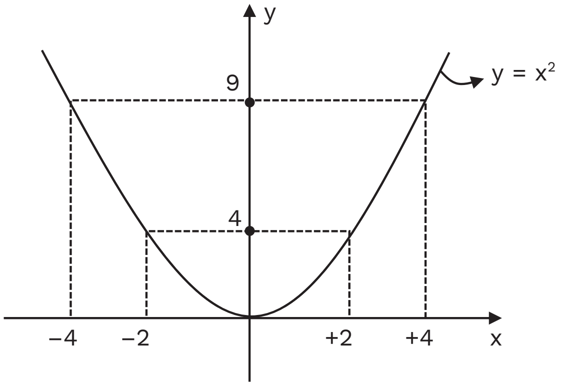
$\Rightarrow x^2 = 4$

$\Rightarrow x_1 = +2, x_2 = -2.$

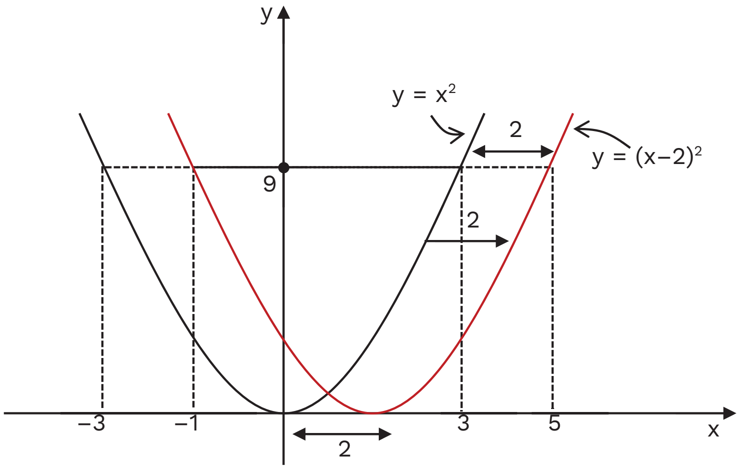
When y = 9

$\Rightarrow x^2 = 9$

$\Rightarrow x_1 = +3, x_2 = -3$



Origin shifting :



$y = x^2$ or $x^2 - y = 0$

Now the graph is pulled towards right by 2 units.

Initially y was zero at x = 0

But now y is zero at x = 2

Everywhere x will be replaced by (x – 2)

New equation ⇒ $y = (x - 2)^2$

Initial equation: $y = x^2$

$y = 9 \rightarrow \begin{array}{|l} x_1 = +3 \\ x_2 = -3 \end{array}$

New equation: $y = (x - 2)^2$

$y = 9 \rightarrow \begin{array}{|l} x_1 = +5 \\ x_2 = -1 \end{array}$

- Similarly, if curve is shifted by 'a' unit in left direction. Then new equation of parabola is $y = (x + a)^2$
- $y = x^2$ parabola is shifted downwards (towards negative y) by 2 units.

$y = x^2$,

replace y with y + 2

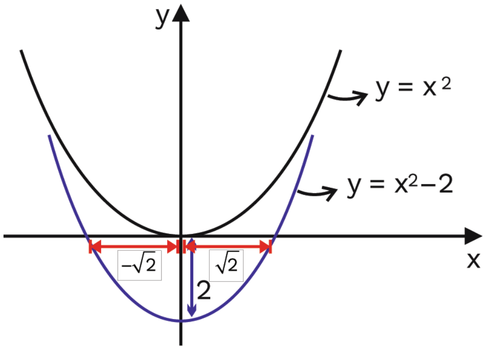
$\therefore \quad (y + 2) = x^2$

$y = x^2 - 2$

$y = 0$

For $y = x^2$

$x^2 = 0 \Rightarrow x = 0$



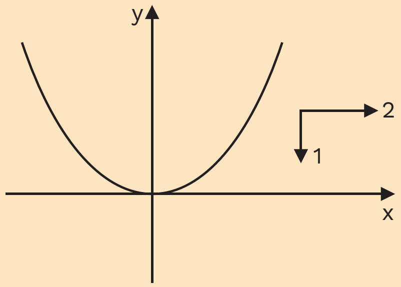
For $y = x^2 - 2$

$\Rightarrow x^2 - 2 = 0$

$x_1 = +\sqrt{2},\ x_2 = -\sqrt{2}$

- Similarly, if parabola $y = x^2$ is pulled in upward direction (+y) by 'b' units, then new equation of parabola is $(y - b) = x^2 \Rightarrow y = x^2 + b$

Q. **$y = x^2$ is pulled by 2 units towards +x direction and 1 unit toward –y direction. Find new equation of parabola.**



Sol.

$$y = x^2 \xrightarrow[\substack{\longrightarrow\\ 2}]{} y = (x-2)^2 \xrightarrow[\downarrow 1]{} y + 1 = (x-2)^2$$

$y + 1 = (x - 2)^2$

$\Rightarrow \quad y + 1 = x^2 + 4 - 4x$

$\Rightarrow \quad y = x^2 - 4x + 3$

$y = x^2 - 4x + 3 \rightarrow$ Parabola

Checking for $y = 0$

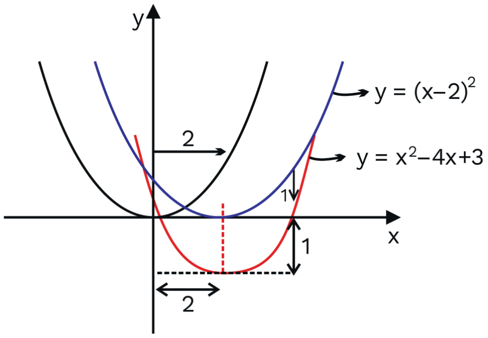
$\Rightarrow \quad x^2 - 4x + 3 = 0 \rightarrow$ Quadratic Equation

$\Rightarrow \quad x^2 - x - 3x + 3 = 0$

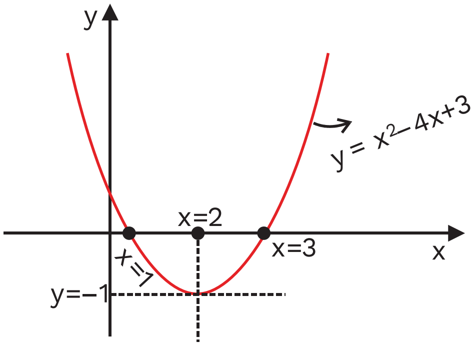
$\Rightarrow \quad x(x - 1) - 3(x - 1) = 0$

$\Rightarrow \quad (x - 1)(x - 3) = 0$

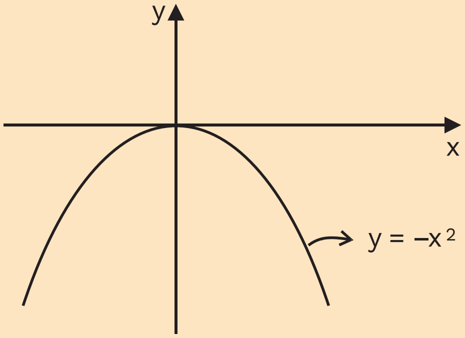
$\Rightarrow \quad x = 1$ and $x = 3$



Exact Graph ⇒

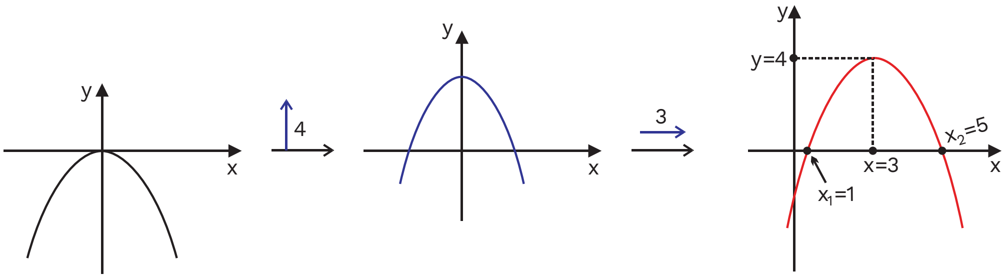


Q. **$y = x^2$**



Now if above curve is pulled upwards by 4 units and towards right by 3 units, draw the new graph.

Sol. $y = -x^2 \xrightarrow{\uparrow 4} y - 4 = -x^2 \xrightarrow{\overrightarrow{3}} y - 4 = -(x-3)^2$



$\Rightarrow\ y - 4 = -(x - 3)^2 \qquad \Rightarrow\ y = 4 - (x^2 + 9 - 6x) \qquad \Rightarrow\ y = -x^2 + 6x - 5$

$y = 0 \Rightarrow -x^2 + 6x - 5 = 0 \qquad \Rightarrow\ x^2 - 6x + 5 = 0 \qquad \Rightarrow\ x^2 - 5x - x + 5 = 0$

$\Rightarrow\ x(x - 5) - 1(x - 5) = 0 \qquad \Rightarrow\ (x - 5)(x - 1) = 0 \qquad x_1 = 1,\ \ x_2 = 5$

Roots of quadratic equation (x_1, x_2) represent intersection of parabola with x-axis.

Binomial Theorem

As per bionomial theorem, we can expand (a + b)^n as,

$$(a+b)^n = (1)a^n\,b^0 + (n)a^{n-1}\,b^1 + \frac{n(n-1)}{1\times 2}a^{n-2}\,b^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}a^{n-3}\,b^3 + \frac{n(n-1)(n-2)(n-3)}{1\times 2\times 3\times 4}a^{n-4}b^4 + \dots$$

After few terms coefficient of $a^{()}\,b^{()}$ will become zero, then stop writing next terms.

For example :

$$(a+b)^2 = 1a^2\,b^0 + 2\,a^{2-1}\,b^1 + \frac{2(2-1)}{1\times 2}a^{2-2}\,b^2 + \frac{2(2-1)(\cancelto{0}{2-2})}{1\times 2\times 3}a^{2-3}b^3$$

Here, n = 2

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

$$(a+b)^3 = 1a^3b^0 + 3\,a^{3-1}\,b^1 + \frac{3(3-1)}{2}a^{3-2}\,b^2 + \frac{3(3-1)(3-2)}{2\times 3}a^{3-3}\,b^3 + \frac{3(3-1)(3-2)(3-3)}{2\times 3\times 4}a^{3-4}\,b^4$$

Here, n = 3

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

Similarly, expansion of $(a+b)^4$, $(a+5)^5$, $(a+b)^6, \dots$ can be written.

- $(99)^6 = (100-1)^6 = (a+b)^n = \dots\dots\dots\dots$

  a = 100

  b = – 1

  n = 6

- $(1+x)^n = 1^n x^0 + n\,1^{(n-1)}x^1 + \frac{n(n-1)}{2}1^{(n-2)}x^2 + \frac{n(n-1)(n-2)}{2\times 3}1^{(n-3)}x^3 + \dots$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{2\times 3}x^3 + \dots$$

Now, suppose ‘x’ is very small ($x \to 0$)

$\Rightarrow$ x^2 is very very small

$\Rightarrow$ x^3 is very very very small

Ignoring very small terms

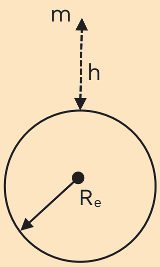
$\boxed{(1+x)^n = 1+nx}$ $\Leftarrow$ Binomial approximation.

Q. **In gravitation, approximae the force on m.**

Mass of earth = m_e

Radius of earth R_e = 6400 km

h = 10m



Sol. $\left(\frac{h}{R_e}\right)$ is very small

Force acting on particle of mass m, $F = \frac{Gm_em}{(R_e+h)^2} = \frac{Gm_em}{R_e^2\left(1+\frac{h}{R_e}\right)^2}$

$$F = \frac{Gm_em}{R_e^2}\left(1+\frac{h}{R_e}\right)^{-2} \quad = \frac{Gm_em}{R_e^2}\left[1+(-2)\frac{h}{R_e}\right], \text{ (Here, x = –2)}$$

$$F = \frac{Gm_em}{R_e^2}\left[1-\frac{2h}{R_e}\right]$$

Q. **Find the value of $(0.99)^2$**

Sol. $(0.99)^2 = (1-0.01)^2 = \left(1-\frac{1}{100}\right)^2 = (1-x)^n$

$[1+(-x)]^n = 1+n(-x) = 1-nx$

$(0.99)^2 = 1-2\times\frac{1}{100} = 1-\frac{2}{100} = \frac{98}{100} = 0.98$

Q. **Find the value of $(0.99)^7$**

Sol. $(0.99)^7 = \left(1-\frac{1}{100}\right)^7 = 1-7\cdot\left(\frac{1}{100}\right) = 1-\frac{7}{100} = \frac{93}{100} \simeq 0.93$

Logarithm

A logarithm is the power to which a number must be raised in order to get some other number e.g. The base ten logarithm of 100 is 2 because ten raised to the power of two is 100.

$$2^4 = 16 \Rightarrow \log_2 16 = 4$$

$$5^2 = 25 \Rightarrow \log_5 25 = 2$$

$$\boxed{a^b = c \Rightarrow \log_a c = b}$$

$$10^1 = 10 \Rightarrow \log_{10} 10 = 1$$

$$100,000 = 10^5 \Rightarrow \log_{10} 100,000 = 5$$

$$\log_{10} 1000 = 3$$

$$\log_4 64 = 3$$

$$\log_2 64 = 6$$

Rules :

$$\log (m \times n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log m^n = n \log m$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Examples :

a. $\log_4 64 = 3$

b. $\log_4 64 = \frac{\log_2 64}{\log_2 4} = \frac{6}{2} = 3$

c. $\log_4 64 = \frac{\log_8 64}{\log_8 4} = \frac{2}{(2/3)} = 3$

d. $\log_8 4 = x \Rightarrow (8)^x = 4 \Rightarrow (8)^{2/3} = 4$

$\therefore \quad x = \frac{2}{3} = \log_8 4$

Q. **Find the value of $\log_{10}(5000)$**

Sol. $\log_{10}(5000) = \log_{10}(5 \times 1000)$

$= \log_{10} 5 + \log_{10} 1000$

$\log_{10}(5000) = 0.6989 + 3 = 3.6989$

$\Rightarrow\ 10^{3.6989} = 5000$

Values to remember :

$\log_{10} 2 = 0.3010,\ \log_{10} 3 = 0.4771,\ \log_{10} 5 = 0.6989, \log_{10} 7 = 0.8450$

Q. **Find the value of $\log_{10}(2500)$**

Sol. $\log_{10} 2500 = \log_{10}(5 \times 5 \times 100)$

$= \log_{10} 5 + \log_{10} 5 + \log_{10} 100$

$= 0.6989 + 0.6989 + 2$

$= 3.3978$

Q. **$\log_e 2500$ = ??**

Sol. e = 2.7 $(2.7)^x = 2500 \ \Rightarrow x = ?$

$\log_{2.7}(2500) = ??$

$\boxed{\log_e M = 2.303 \log_{10} M}$

$\log_e 2500 = 2.303 \times \log_{10} 2500$

$\log_e 2500 = 2.303 \times 3.3978$

Examples :

a. $\log_{10} 25 = \log_{10} 5^2$

$= 2\log_{10} 5 \qquad [\log m^n = n\log m]$

$= 2 \times 0.6989$

b. $\log_{10} 42 = \log_{10}(2 \times 3 \times 7)$

$= \log_{10} 2 + \log_{10} 3 + \log_{10} 7$

$= 0.3010 + 0.4771 + 0.8450 = 1.6321$

c. $\log_e 42 = 2.303 \log_{10} 42 = 2.303 \times 1.6321$

d. $\log_5 42 = \dfrac{\log_{10} 42}{\log_5 42} = \dfrac{1.6321}{0.6989}$

e. $\log_{10} 1/8 = \log_{10}\left(\dfrac{1}{2^3}\right) = \log_{10} 2^{-3} = -3 \log_{10} 2$

$= -3 \times 0.3010 = -0.9030$

f. $\log_{10} \sqrt{24} = \dfrac{1}{2}\log_{10} 24 = \dfrac{1}{2}\log_{10}(2 \times 3 \times 2 \times 2)$

$= \dfrac{1}{2}\log_{10}(2^3 \times 3) = \dfrac{1}{2}[3\log_{10} 2 + \log_{10} 3] = \dfrac{1}{2}[3 \times 0.3010 + 0.4771] = 0.6901$

g. $\log_{10}\left(\dfrac{1}{2}\right) = -\log_{10} 2 = -0.3010$

h. $\log_e\left(\dfrac{1}{2}\right) = -2.303 \log_{10} 2 = -0.693$

Series

Arithmetic Progression (A.P.)

A series of the form of,

$$\underset{\substack{1^{st}\\ \text{term}}}{a}\ ,\ \underset{\substack{2^{nd}\\ \text{term}}}{a+d},\ \underset{\substack{3^{rd}\\ \text{term}}}{a+2d},\ \underset{\substack{4^{th}\\ \text{term}}}{a+3d},\ \underset{\substack{5^{th}\\ \text{term}}}{a+4d}\ \ldots\ldots\ldots\ldots,\ \underset{\substack{n^{th}\\ \text{term}}}{a+(n-1)d},\ldots\ldots\ldots\ldots$$

is called an arithmetic progression.

where, $d = (2^{nd}\text{term}) - (1^{st}\text{term}) = (3^{rd}\text{term} - 2^{nd}\text{term}) = \ldots\ldots,$ is known as common difference.

For example :

3, 5, 7, 9, 11,......

1^{st} term, a = 3

2^{nd} term, a + d = 5

Common difference = 5 – 3 = 2

We can find n^{th} term as,

n^{th} term = a + (n – 1)d = 3 + (n – 1)2

4^{th} term = 3 + (4 – 1)2 = 3 + 6 = 9

5^{th} term = 3 + (5 – 1)2 = 3 + 8 = 11

1, 2, 3, 4, 5,......., n,......

Sum of n terms, $S = \frac{n}{2}[2a + (n-1)d]$

Q. **For the given A.P,**
3, 5, 7, 9, 11 , ...
Calculate sum upto 4 terms

Sol. $S = \frac{4}{2}[2 \times 3 + (4-1)2] = 2[6 + 6] = 24$

Check : 3 + 5 + 7 + 9 = 24

Geometric progression (G.P) :

A series of the form,

$$\underset{\substack{1^{st}\\ \text{term}}}{a}\,,\ \underset{\substack{2\,nd\\ \text{term}}}{ar}\,,\ \underset{\substack{3\,rd\\ \text{term}}}{ar^2}\,,\ \underset{\substack{4\,th\\ \text{term}}}{ar^3}\ \ ....,\ \underset{\substack{n\,th\\ \text{term}}}{ar^{n-1}},..........., \text{ is called a geometirc progression (G.P.)}$$

$$\text{Where, common ratio } = \frac{2^{nd}\text{term}}{1^{st}\text{term}} = \frac{3^{rd}\text{term}}{2^{nd}\text{term}} = ... = \frac{n^{th}\text{term}}{(n-1)^{th}\text{term}} = r$$

For example :

If 1^{st} term, $a = 4$ & Common ratio, $r = \frac{1}{2}$

then G.P. is $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{5},........$

We can find n^{th} term of an G.P. as,

$$n^{th} \text{ term } = a \cdot r^{n-1} = 4\left(\frac{1}{2}\right)^{n-1}$$

$$\text{e.g., } 5^{th} \text{ term } = 4\left(\frac{1}{2}\right)^{5-1} = 4\left(\frac{1}{2}\right)^{4} = \frac{4}{16} = \frac{1}{4}$$

Sum of n terms of G.P :

$$\boxed{S = \frac{a(1-r^n)}{1-r}}$$

- For G.P. series having infinite number of terms

If $r < 1,\ S_\infty = \frac{a}{1-r}$

If $r > 1,\ S_\infty = \infty$

Q. **4, 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$,.........∞ . Find sum.**

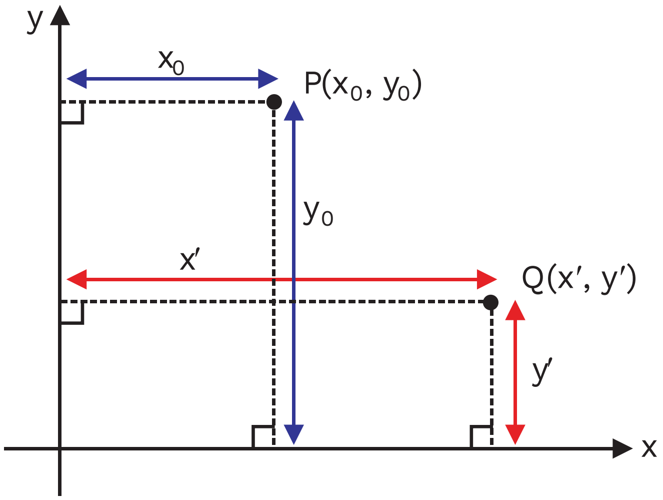
Sol. Here $r < 1$ for this ∞ series $\left(r = \frac{1}{2}\right)$

$$\text{So, } S = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = 8$$

$S = 8$

Co-Ordinate Geometry

Let us consider two points P and Q in XY plane as shown.



x-coordinate of point P = x_0 = Perpendicular distance of ‘P’ from y-axis

y-coordinate of point P = y_0 = Perpendicular distance of ‘P’ from x-axis

x-coordinate of point Q = x' = Perpendicular distance of ‘Q’ from y-axis

y-coordinate of point Q = y' = Perpendicular distance of ‘Q’ from x-axis

Distance Formula :

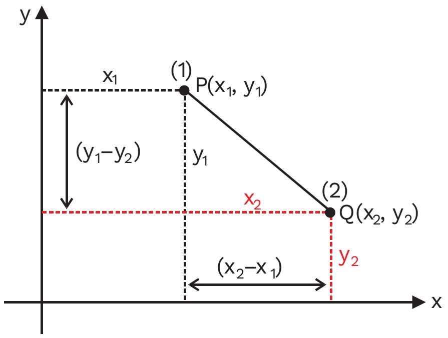
Distance between points P and Q

$$PQ = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

∴ Distance between two points having

co-ordinates (x_1, y_1) and (x_2, y_2) is given by

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Straight Line :

Equation of straight line $\Rightarrow \boxed{y = mx + c}$

c = Intercept on y-axis

m = slope = tan θ

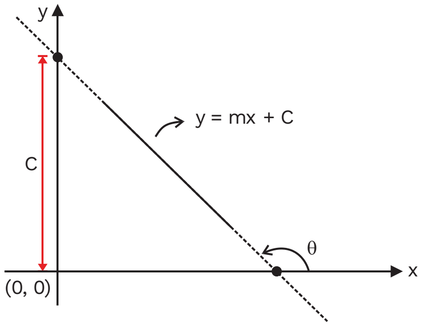
where 'θ' is angle made by line with +ve x-axis.

If m is positive

$$\Rightarrow \begin{cases} \text{x increases then y also increases} \\ \text{x decreases then y also decreases} \end{cases}$$

If m is negative

$$\Rightarrow \begin{cases} \text{x increases then y decreases} \\ \text{x decreases then y increases} \end{cases}$$



For example :

Case a.

Here C is positive when moving from A → B,

x increases but y decreases ⇒ $m < 0$

When moving from B → A,

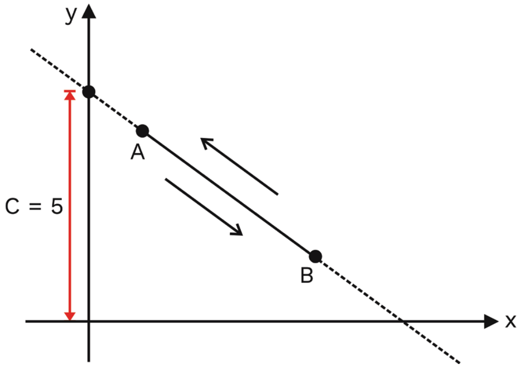
x decreases but y increases ⇒ $m < 0$

∴ Slope of given line is negative.

∴ Equation of given line could be

$\boxed{y = -2x + 5}$

m = –2 (–ve) ; C = +5 (+ve)



Case b.

Here C is negative

When moving from A → B

x → increases and y → increases

⇒ $m > 0$

When moving from B → A

x → decreases and y → decreases

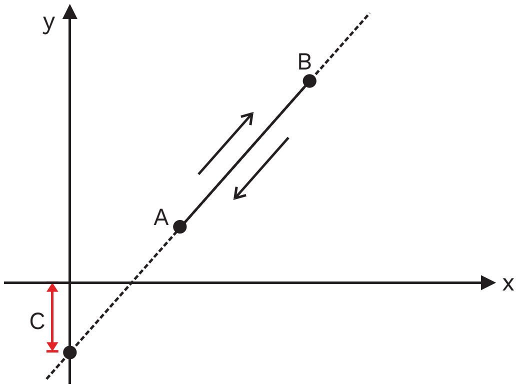
⇒ $m > 0$

∴ Equation of given line could be

$\boxed{y = +3x - 2}$

m = +3 (+ve)

c = –2 (–ve)



- **How to write the equation of given line ?**

Let, the equation of the given line is, y = mx + c

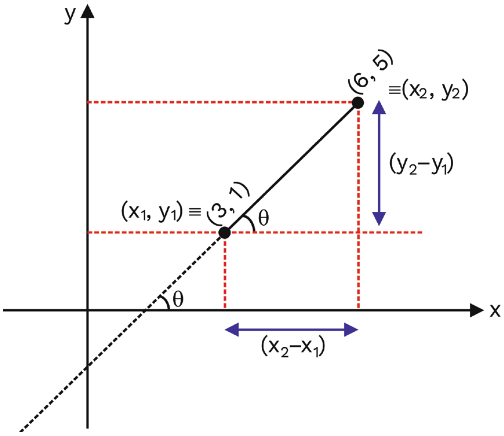
$$\boxed{\text{Slope} = m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}}$$

See the triangle

$$\therefore \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{6-3} = \frac{4}{3}$$

$$m = \frac{4}{3}$$

$$\therefore \quad y = \frac{4}{3}x + C$$



As point (3, 1) lies on straight line, it must satisfy the equation of straight line i.e.

When x = 3, we get y = 1

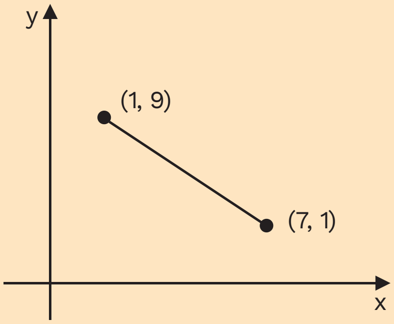
$$1 = \frac{4}{3}\cdot 3 + C$$

$$\Rightarrow \quad C = 1 - 4 = -3$$

$$\Rightarrow \quad y = \frac{4}{3}x - 3$$

[Point (6, 5) will also satisfy the equation of straight line]

Q. **Find equation of straight line**



Sol. Let, $(x_2, y_2) = (1, 9)$ and $(x_1, y_1) = (7, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{1-7} = \frac{8}{-6} = \frac{-4}{3}$$

$$y = mx + c$$

$$y = \frac{-4}{3}x + C$$

Now, Point (1, 9) will satisfy this equation.

$$\Rightarrow\ 9 = \frac{-4}{3}\cdot 1 + C \ \Rightarrow\ C = 9 + \frac{4}{3} = \frac{31}{3}$$

$$\Rightarrow\ \boxed{y = \frac{-4}{3}x + \frac{31}{3}} \ \Rightarrow\ 3y = -4x + 31$$

- Another way to write equation of straight line is

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

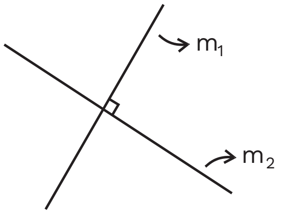
Here, $x_1 = 7$, $y_1 = 1$, $x_2 = 1$, $y_2 = 9$

$$\Rightarrow\ y - 1 = \frac{9-1}{1-7}(x-7) \ \Rightarrow\ y - 1 = \frac{-4}{3}(x-7)$$

$$\Rightarrow\ 3y - 3 = -4x + 28 \Rightarrow \boxed{3y + 4x = 31}$$

- If two lines having slopes m_1 and m_2 are perpendicular to each other, then

$$\boxed{m_1 \cdot m_2 = -1}$$



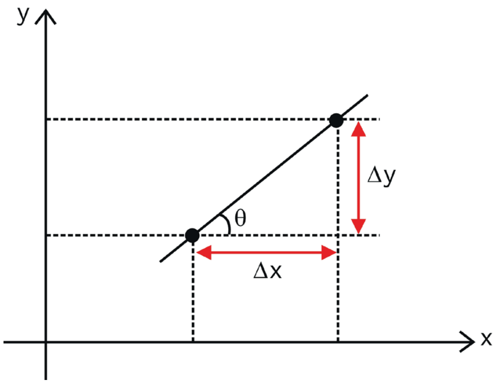
$$\boxed{m_1 \cdot m_2 = -1}$$

- Slope $= m = \tan\theta = \dfrac{\Delta y}{\Delta x}$

  Δ = Change

  = final value – Initial value

  Slope $= \dfrac{\Delta y}{\Delta x}$ = Rate of change of y w.r.t x

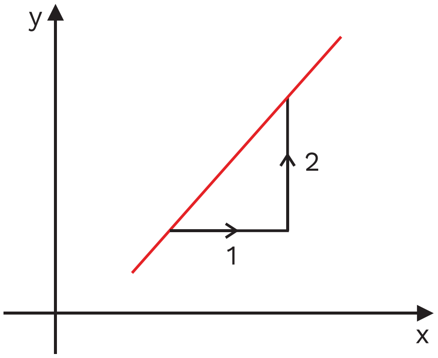


For example :

If Slope = 2

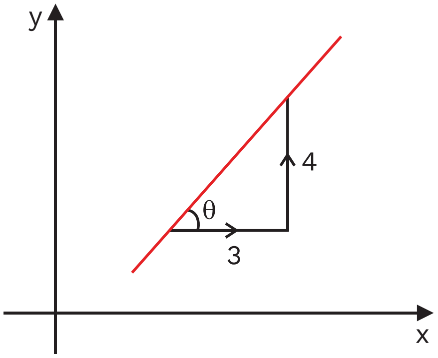
then, $\frac{\Delta y}{\Delta x} = 2 \Rightarrow \quad \Delta y = 2\Delta x$

If x increases by 1 then y increases by 2



For slop, $m = \frac{4}{3} = \tan\theta = \frac{\Delta y}{\Delta x}$

$\Delta y = \frac{4}{3}\Delta x$



- If Δx is very small then

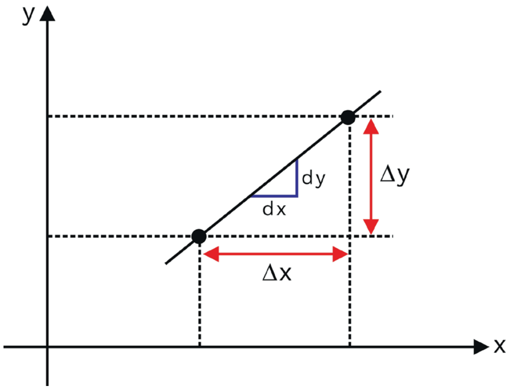
  Slope $= m = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

  $\boxed{\frac{dy}{dx} = \text{differentiation}}$

- $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = m = \text{Slope} = \tan\theta = \frac{dy}{dx}$

  = Rate of change of y w.r.t x

  = Differentiation of y w.r.t x



Differentiation

FUNCTION

A function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output.

e.g., $y = t^2 + t$

On changing 't', y changes

$\therefore$ y is a function of 't' $\Rightarrow$ $y = f(t)$

Conversely, on changing y, t changes

$\therefore$ t is a function of y $\Rightarrow$ $t = g(y)$

- $y = x^2$

  $\Rightarrow$ $y = f(x)$ and $x = g(y)$

- $y = \sin x$

  $y = f(x)$

- $y = x^2 + x$ and $x = 2t$

  $y = x^2 + x = (2t)^2 + (2t) = 4t^2 + 2t$

  Here, $y = f(x)$ and $x = g(t)$

  Combining the two relations,

  $y = f(x) = f[g(t)]$ $[\because x = g(t)]$

  $y = f[g(t)] = fog(t) \leftarrow$ Composite function

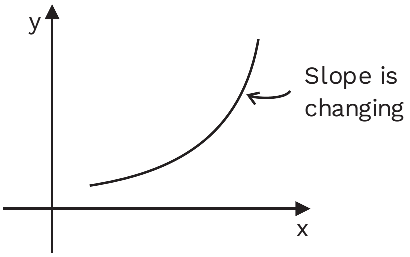
DIFFERENTIAL CALCULUS

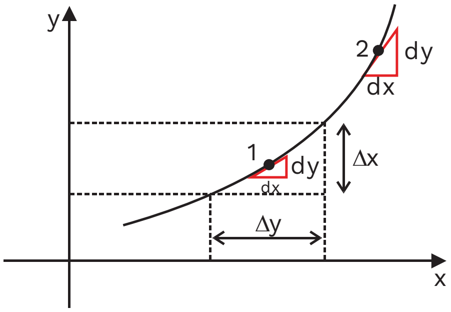
We know that, $\text{slope} = \tan\theta = \dfrac{\Delta y}{\Delta x}$ [when slope is constant]

and, $\text{slope} = \tan\theta = \dfrac{dy}{dx}$ [when slope changes]

Now, $\dfrac{dy}{dx}$ = Rate of change of 'y' w.r.t 'x'

= Differentiation of 'y' w.r.t 'x'





- Slope at point (1)

We can't get correct value of slope by taking big interval for a curved graph.

$\therefore \quad \frac{\Delta y}{\Delta x} \neq$ correct slope

$\frac{dy}{dx}$ = more correct slope (for very small interval)

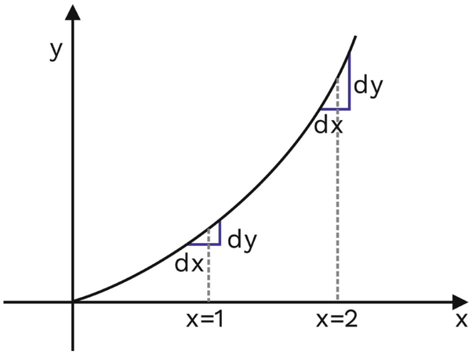
Similarly, by calculating $\frac{dy}{dx}$ at point (2), we can get slope at point 2

- Changing slope = curve

Constant slope = straight line

- $y = x^2$, calculate slope at x = 1 and x = 2

$$\left(\frac{dy}{dx}\right)_{x=1} < \left(\frac{dy}{dx}\right)_{x=2}$$



It is difficult to find slope $\left(\frac{dy}{dx}\right)$ by drawing graph for every function. To ease it, Newton developed formulae in calculus book. We will use those formulae to calculate $\frac{'dy'}{dx}$ for functions whose graphs cannot be drawn easily.

Functions	Differentiation	Examples
$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$	$y = x^7 \rightarrow \frac{dy}{dx} = 7x^{7-1} = 7x^6$
$y = x$	$\frac{dy}{dx} = 1$	$y = x^1 \rightarrow \frac{dy}{dx} = 1 \cdot x^{1-1} = 1 \cdot x^0 = 1$
$y = ax$	$\frac{dy}{dx} = a$	$y = 7x \rightarrow \frac{dy}{dx} = 7 \cdot \frac{dx}{dx} = 7 \cdot 1 = 7$
$y = \sin x$	$\frac{dy}{dx} = \cos x$	
$y = \cos x$	$\frac{dy}{dx} = -\sin x$	
$y = \tan x$	$\frac{dy}{dx} = \sec^2 x$	
$y = \operatorname{cosec} x$	$\frac{dy}{dx} = -\operatorname{cosec} x \cdot \cot x$	
$y = \sec x$	$\frac{dy}{dx} = \sec x \tan x$	
$y = \cot x$	$\frac{dy}{dx} = -\operatorname{cosec}^2 x$	
$y = a^x$ a=constant	$\frac{dy}{dx} = a^x \cdot \log_e a$	$y = 7^x \rightarrow \frac{dy}{dx} = 7^x \cdot \log_e 7 = 7^x \ell n 7,$ $e \simeq 2.7$
$y = e^x$	$\frac{dy}{dx} = e^x$	$y = e^x \rightarrow \frac{dy}{dx} = e^x \log_e e = e^x \cdot 1 = e^x$
$y = \log_e x = \ell nx$	$\frac{dy}{dx} = \frac{1}{x}$	
$y = \sin^{-1} x$	$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$	
$y = \sec^{-1} x$	$\frac{dy}{dx} = \frac{1}{\lvert x \rvert \sqrt{x^2 - 1}}$	
$y = \tan^{-1} x$	$\frac{dy}{dx} = \frac{1}{1 + x^2}$	

Rules :

- y = const. $\Rightarrow \frac{dy}{dx} = 0$
- y = x $\Rightarrow \frac{dy}{dx} = 1$
- y = u + v, where u = f(x) and v = g(x)
- $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

For example,

a. $y = x^2 + \sin x$

Here $u = x^2$, $v = \sin x$

then, $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = 2x + \cos x$

b. $y = c \times u$, where c = constant and u = f(x)

then, $\frac{dy}{dx} = c \cdot \frac{du}{dx}$

c. y = 3 sinx

then, $\frac{dy}{dx} = 3 \cdot \frac{d}{dx}(\sin x) = 3\cos x$

Product Rule :

For $y = u \times v$, where u = f(x) and, v = g(x)

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

For example,

If $y = x^2 \sin x$

then, $\frac{dy}{dx} = x^2 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^2)$

$\frac{dy}{dx} = x^2 \cdot \cos x + \sin x \cdot 2x$ = x[xcosx + 2sinx]

Quotient Rule :

- $y = \frac{u}{v}$, where u = f(x) and, v = g(x)

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

For example,

If $y = \frac{\sin x}{x^2}$,

then, $\frac{dy}{dx} = \frac{x^2 \cdot \cos x - \sin x \cdot 2x}{x^4}$

Q. **If $y = \frac{e^x}{x^3}$, then find $\frac{dy}{dx}$**

Sol. $$\frac{dy}{dx} = \frac{x^3 \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x^3)}{(x^3)^2} = \frac{x^3 \cdot e^x - e^x \cdot 3x^2}{x^6} = \frac{x^2(x-3)e^x}{x^6}$$

$$\frac{dy}{dx} = \frac{(x-3)e^x}{x^4}$$

Q. **If $y = x^3 \cdot \tan x$, then find $\frac{dy}{dx}$**

Sol. $$\frac{dy}{dx} = x^3 \cdot \frac{d}{dx}(\tan x) + \tan x \cdot \frac{d}{dx}(x^3)$$

$$= x^3 \cdot \sec^2 x + \tan x \cdot (3x^2)$$

$$= x^2(x \sec^2 x + 3 \tan x)$$

Q. **If $y = \sqrt{2x}$, $\frac{dy}{dx} = ?$**

Sol. $y = \sqrt{2x} = \sqrt{2} \cdot x^{1/2}$ (Here, n = 1/2)

$$\frac{dy}{dx} = \sqrt{2} \frac{d}{dx}(x^{1/2}) = \sqrt{2} \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{\sqrt{2}}{2} \cdot x^{\frac{-1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2x}}$$

Chain Rule :

In calculus, the chain rule is a formula to compute the derivative of a composite function.

→ $y = 4\sin(3x)$

$$\frac{dy}{dx} = ?$$

Let, $t = 3x$ ...(1) $\Rightarrow \frac{dt}{dx} = 3$

$y = 4\sin t$ ...(2) $\Rightarrow \frac{dy}{dt} = 4\frac{d}{dt}(\sin t) = 4\cos t$

$$\frac{dy}{dt} * \frac{dt}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 4\cos t \cdot 3 = 12\cos t \Rightarrow \frac{dy}{dx} = 12\cos(3x)$$

Q. $y = 4e^{x^2-2x}$ $\left[y = e^x, \frac{dy}{dx} = e^x\right]$

Find $\frac{dy}{dx} = ?$

Sol. $t = x^2 - 2x \Rightarrow \frac{dt}{dx} = 2x - 2$

$y = 4e^t \Rightarrow \frac{dy}{dt} = 4 \cdot e^t$

$$\frac{dy}{dx} = \frac{dy}{dt} * \frac{dt}{dx}$$

$$= 4e^t \cdot (2x - 2)$$

$$= 4e^{x^2-2x} \cdot (2x - 2)$$

Q. **y = $\ell n(\cos 3x)$, find $\frac{dy}{dx}$.**

Sol. $P = 3x$ ...(i) $\Rightarrow \frac{dP}{dx} = 3$

$t = \cos 3x = \cos P$ ...(ii) $\Rightarrow \frac{dt}{dP} = -\sin P$

$y = \ell nt$ ...(iii) $\Rightarrow \frac{dy}{dt} = \frac{1}{t}$

$$\underbrace{\frac{dy}{dt} \times \frac{dt}{dP} \times \frac{dP}{dx}}_{\text{Chain}} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{t} \times 3 \times (-\sin P)$$

$$= \frac{1}{\cos 3x} \cdot 3 \cdot (-\sin 3x)$$

$$\frac{dy}{dx} = -3\tan 3x$$

Q. **y = $\sqrt{\log_e x} = (\log_e x)^{1/2}$. Find $\frac{dy}{dx}$**

Sol. $t = \log_e x \Rightarrow \frac{dt}{dx} = \frac{1}{x}$

$y = t^{1/2} \Rightarrow \frac{dy}{dt} = \frac{1}{2} \cdot t^{-1/2} = \frac{1}{2\sqrt{t}}$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{2\sqrt{t}} \cdot \frac{1}{x}$$

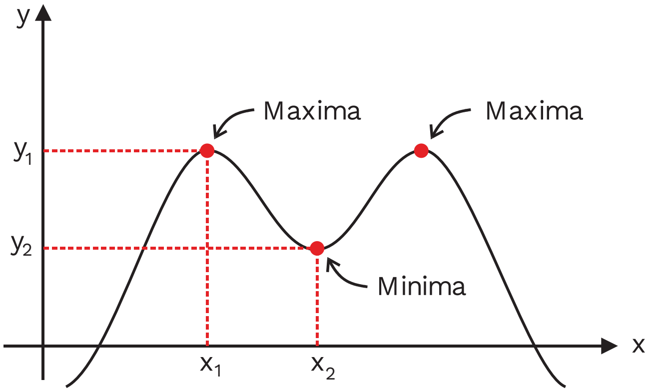
$$\frac{dy}{dx} = \frac{1}{2\sqrt{\log_e x}} \cdot \frac{1}{x}$$

Maxima-Minima :

y = f(x)

Maxima : Value of y at that point is greater than values of y at immediate points in the left and right.

Minima : Value of y at minima is less than values of y at immediate points in the left and right.



There can be many number of maxima and minima in a single curve. That is why it is also called as local maxima and local minima.

If on increasing x, y also increases.

$\Rightarrow$ dx > 0 then dy > 0

$$\Rightarrow \frac{dy}{dx} > 0$$

or If on decreasing x, y also decreases.

$\Rightarrow$ dx < 0 then dy < 0

$$\Rightarrow \frac{dy}{dx} > 0$$

Similarly, If on increasing x, y decreases or on decreasing x, y increases

$\Rightarrow$ dx > 0 then dy < 0 or dx < 0 then dy > 0

$$\Rightarrow \frac{dy}{dx} < 0$$

Maxima :

At maxima, slope, m = 0

$$\boxed{\frac{dy}{dx}=0} \qquad \text{...(1)}$$

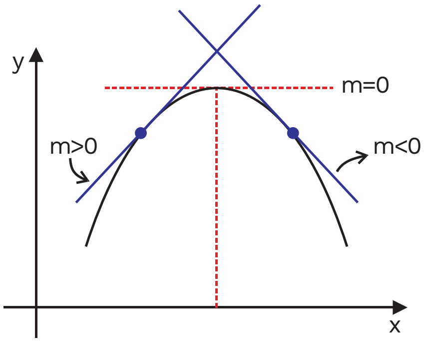
$\frac{d^2y}{dx^2}$ = Double differentiation

Also as x increases, m decreases

(+ve → 0 → −ve)

$$\therefore \quad \frac{dm}{dx}<0,\ m=\frac{dy}{dx} \ \Rightarrow \ \frac{d}{dx}\left(\frac{dy}{dx}\right)<0$$

$$\Rightarrow \ \boxed{\frac{d^2y}{dx^2}<0} \qquad \text{...(2)}$$



Minima :

At minima, slope m = 0

$$\boxed{\frac{dy}{dx}=0} \qquad \text{...(1)}$$

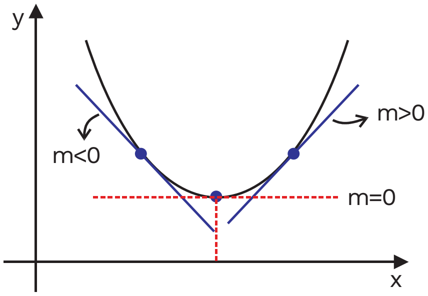
Also, as x increases, m increases

$(-v_e \rightarrow 0 \rightarrow +v_e)$

$$\therefore \quad \frac{dm}{dx}>0,\ m=\frac{dy}{dx}$$

$$\Rightarrow \ \frac{d}{dx}\left(\frac{dy}{dx}\right)>0$$

$$\Rightarrow \ \boxed{\frac{d^2y}{dx^2}>0} \qquad \text{...(2)}$$



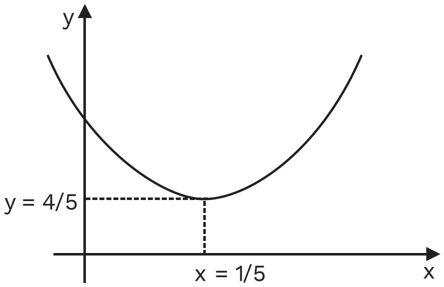
Q. **$y = 5x^2 - 2x + 1$, find maxima or minima.**

Sol.

$$m = \frac{dy}{dx} = 10x - 2$$

At maxima/minima, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 10x - 2 = 0 \quad \Rightarrow \quad x = \frac{2}{10} = \frac{1}{5}$$



At $x = \frac{1}{5}$ there can be maxima or minima.

$$\frac{dm}{dx} = \frac{d^2y}{dx^2} = +10$$

As $\frac{d^2y}{dx^2} > 0$,

So, at $x = \frac{1}{5}$, there is a minima.

$$y_{min} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) + 1 \Rightarrow y = \frac{4}{5}$$

Q. **For $y = x^3 - 3x^2$, find max/min.**

Sol.

$$\frac{dy}{dx} = 3x^2 - 6x$$

At maxima or minima, $\frac{dy}{dx} = 0$

$$\Rightarrow \quad 3x^2 - 6x = 0$$

$$\Rightarrow \quad 3x(x-2) = 0 \quad \left| \begin{array}{l} x = 0 \rightarrow \text{max/ min} \\ x = 2 \rightarrow \text{max/ min} \end{array} \right.$$

Let's check for maxima or minima.

$$\frac{dm}{dx} = \frac{d^2y}{dx^2} = 6x - 6$$

At $x = 0 \quad \Rightarrow \frac{d^2y}{dx^2} = -6$

As $\frac{d^2y}{dx^2} < 0$ at $x = 0$

So, at x = 0, there is a maxima.

$y_{max} = 0^3 - 3(0)^2 = 0$

At x = 2, $\frac{d^2y}{dx^2} = +6$

So, at x = 2, $\frac{d^2y}{dx^2} > 0$

∴ At x = 2, there is a minima

ymin = $2^3 - 3(2)^2 = -4$

Q. **y = sinx + cos(2x), find $\frac{d^2y}{dx^2}$.**

Sol. For y = sinx + cos(2x)

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos 2x) = \cos x + \underbrace{\frac{d}{dx}(\cos 2x)}_{\text{Chain Rule}}$$

Let z = cos2x

and, $t = 2x \rightarrow \frac{dt}{dx} = 2$

$$z = \cos t \rightarrow \frac{dz}{dt} = -\sin t$$

$$\frac{d}{dx}(\cos 2x) = \frac{dz}{dx} = \frac{dz}{dt} \cdot \frac{dt}{dx} = -\sin t \cdot 2$$

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x \quad \Rightarrow \quad \frac{dy}{dx} = \cos x - 2\sin 2x$$

Now,

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\cos x) - 2\frac{d}{dx}(\sin 2x)$$

$$\frac{d^2y}{dx^2} = -\sin x - 2\frac{d}{dx}(\sin 2x)$$

Let m = sin2x

and, $t = 2x \rightarrow \frac{dt}{dx} = 2$

$\Rightarrow \ m = \sin t \rightarrow \frac{dm}{dt} = \cos t$

$\Rightarrow \ \frac{d}{dx}(\sin 2x) = \frac{dm}{dx} = \frac{dm}{dt} \times \frac{dt}{dx} = \cos t \times 2 = 2\cos 2x$

$\Rightarrow \ \frac{d^2y}{dx^2} = -\sin x - 4\cos 2x$

Integration

INTEGRAL CALCULUS

Integration is reverse process of differentiation.

$$\begin{tikzcd} y = x^3 \arrow[r, bend left, "\text{Differentiation w.r.t } x", "\frac{dy}{dx}"'] & 3x^2 \arrow[l, bend left, "\text{Integration w.r.t } x"', "\int 3x^2dx"] \end{tikzcd}$$

$$x^3 = \int 3x^2\,dx$$

For example :

$$\int x^7 dx = ?$$

Let, $\int x^7 dx = y$

$$\frac{dy}{dx} = x^7 \qquad \ldots(1)$$

Let, $y = ax^8$

$$\frac{dy}{dx} = 8ax^7 \qquad \ldots(2)$$

Comparing (1) and (2)

$$1 = 8a \quad \Rightarrow a = \frac{1}{8}$$

$$\therefore \quad y = \frac{x^8}{8}$$

$$\int x^7 dx = \frac{x^8}{8}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \Leftarrow \text{'C' is constant}$$

But why this constant

$$y = x^3 + C$$

$$\frac{dy}{dx} = 3x^2 + 0 = 3x^2$$

$$\therefore \quad \int 3x^2 dx = x^3 + C$$

Similarly, $\int x^7 dx = \frac{x^8}{8} + C$

Think that integration is reverse process of differentiation and you can easily predict the results given below.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int 1 dx = x + C$$

$$\int a\, dx = ax + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = +\sin x + C$$

$$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$$

$$\int \sec^2 x\, dx = \tan x + C$$

$$\int \operatorname{cosec}^2 x\, dx = -\cot x + C$$

$$\int \frac{1}{x} dx = \ell n x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\log_e a} + C$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + C$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + C$$

$$\int \frac{1}{|x| \sqrt{x^2 - 1}} dx = \sec^{-1}(x) + C$$

Rules :

- $\int a\, dx = ax + C$
- $\int (u+v)\, dx = \int u\, dx + \int v\, dx$

For example :

$$\int (x^2 + \sin x)\, dx = \int x^2 dx + \int \sin x\, dx = \frac{x^3}{3} - \cos x + C$$

- $\int a \cdot f(x) dx = a \int f(x)\, dx$

For example :

$$\int 3x^3dx = 3\int x^3dx \;= 3\frac{x^4}{4} + C \;= \frac{3}{4}x^4 + C$$

Substitution Method

Q. **Find the integration,**

$$\mathbf{I = \int \sin^3\theta \cdot \cos\theta \, d\theta}$$

Sol. Put $u = \sin\theta \;\Rightarrow\; \frac{du}{d\theta} = \cos\theta \;\Rightarrow\; du = \cos\theta \, d\theta$

$$I = \int u^3 \, du \;= \frac{u^4}{4} + C$$

$$I = \frac{\sin^4\theta}{4} + C$$

Q. **Find the integration**

$$\mathbf{I = \int \frac{\tan^5\theta}{\cos^2\theta} d\theta}$$

Sol. $u = \tan\theta$

$$\frac{du}{d\theta} = \sec^2\theta \Rightarrow \; du = \sec^2\theta \, d\theta$$

$$I = \int \tan^5\theta \cdot \sec^2\theta \, d\theta$$

$$= \int u^5 \cdot du \;= \frac{u^6}{6} + C$$

$$I = \frac{\tan^6\theta}{6} + C$$

Rule :

- $\int \frac{du}{u} = \ell nu + C$

Q. **$I = \int \frac{1}{\sin\theta} \times \cos\theta \, d\theta$, find I.**

Sol. $u = \sin\theta \Rightarrow du = \cos\theta \, d\theta$

$$I = \int \frac{1}{u} \cdot du = \ell nu + C$$

$$I = \ell n \sin\theta + C$$

$$\therefore \quad \int \cot\theta \, d\theta = \ell n \sin\theta + C$$

Q. **Find the integral**

$$I = \int \frac{dx}{5 - 2x}$$

Sol. Put, $t = 5 - 2x$, so that,

$$\Rightarrow \frac{dt}{dx} = -2 \quad \Rightarrow \frac{dt}{-2} = dx$$

$$I = \frac{-1}{2} \int \frac{dt}{t} = -\frac{1}{2} \ell nt + C$$

$$I = -\frac{1}{2} \ell n(5 - 2x)$$

Q. **Evaluate the integral**

$$I = \int \frac{\sqrt{x}}{1 + x\sqrt{x}} dx$$

Sol. Put, $t = 1 + x\sqrt{x} = 1 + x^{3/2}$

So that, $\frac{dt}{dx} = 0 + \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}\sqrt{x} \Rightarrow \frac{2}{3}dt = \sqrt{x}\,dx$

$$I = \frac{2}{3}\int \frac{dt}{t} = \frac{2}{3}\ell nt + C$$

$$I = \frac{2}{3}\ell n(1 + x\sqrt{x}) + C$$

Q. **Evaluate the integral**

$$\mathbf{I = \int \cos^3\theta \sin\theta \, d\theta}$$

Sol. $t = \cos\theta \Rightarrow \frac{dt}{d\theta} = -\sin\theta \Rightarrow dt = -\sin\theta\, d\theta$

$$I = \int t^3(-dt) = -\frac{t^4}{4} + C$$

$$I = \frac{\cos^4\theta}{4} + C$$

Q. **Find integration**

$$\mathbf{I = \int \frac{e^t dt}{\sqrt{1 - e^t}}}$$

Sol. Put $x = 1 - e^t$, so that

$$\frac{dx}{dt} = 0 - e^t \Rightarrow dx = -e^t dt$$

$$I = -\int \frac{dx}{\sqrt{x}} = -\int x^{-1/2}dx$$

$$I = -\left(\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}\right) + C$$

$$I = -2x^{1/2} + C = -2\sqrt{x} + C = -2\sqrt{1 - e^t} + C$$

Definite Integration :

In definite integration, we use limits of integration and get a constant value for the constant of integration (c). For the function, y = x^3,

$$\int_{x=1}^{x=2} y\,dx = \int_1^2 x^3\,dx = \left[\frac{x^4}{4}\right]_{\underbrace{1}_{\text{lower limit}}}^{\overbrace{2}^{\text{upper limit}}} = \left[\left(\frac{2^4}{4}\right)-\left(\frac{1^4}{4}\right)\right] = 4-\frac{1}{4}=\frac{15}{4}$$

Q. **Evaluate the integral,**

$$I = \int_1^2 (2y+1)^7\,dy$$

Sol. Put t = 2y +1

So that, $\frac{dt}{dy} = 2+0 = 2 \Rightarrow \; dy = \frac{dt}{2}$

So, $I = \int t^7 \frac{dt}{2} = \frac{1}{2}\int t^7 dt = \frac{1}{2}\left[\frac{t^8}{8}\right] = \frac{1}{2}\cdot\frac{1}{8}[t^8]$

$$I = \frac{1}{16}\left[(2y+1)^8\right]_1^2$$

$$I = \frac{1}{16}\left[(5^8)-(3^8)\right] = \frac{5^8-3^8}{16} \;\; = 24004$$

Area Under the Curve : (Geometrical Meaning of Integration)

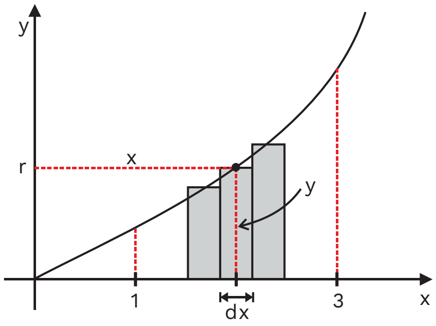
$y = x^2$

$$I = \int_1^3 y\,dx = \int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{1}{3}\left[(3^3) - (1^3)\right]$$

$$I = \frac{1}{3}(26) = \frac{26}{3}$$

$$I = \int_1^3 y\,dx$$

= Area under y vs x curve from x = 1 to x = 3

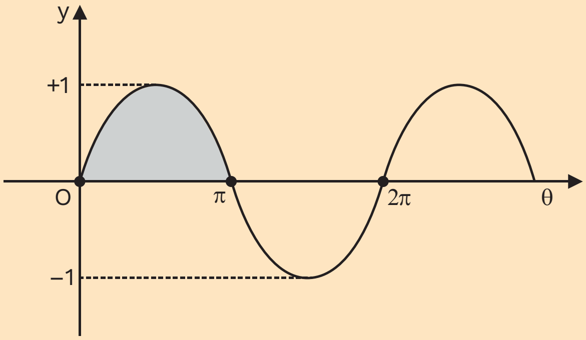


Q. **Find area under $y = x^3 + 3$, with x-axis from $x = 1$ to $x = 3$.**

Sol. $$\text{Area} = \int_1^3 y\,dx = \int_1^3 (x^3 + 3)dx = \left[\frac{x^4}{4}\right]_1^3 + 3[x]_1^3 = \left(\frac{3^4 - 1^4}{4}\right) + 3(3-1) = 26 \text{ units}$$

Q. **For $y = \sin\theta$**

Find area under y vs from $\theta = 0$ to $\theta = \pi$



Sol. $$\text{Area} = \int_0^\pi y\,d\theta = \int_0^\pi \sin\theta\,d\theta$$

$$= -[\cos\theta]_0^\pi = -[\cos\pi - \cos 0] = -[(-1) - (1)]$$

Area = 2 unit

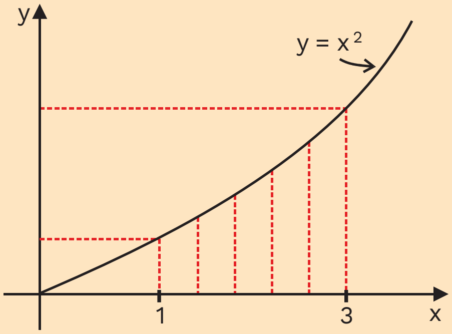
Average Value :

Average value of a function can be found using the relation,

$$\boxed{y_{avg} = \langle y \rangle = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx}}$$

Q. **For y = f(x) = x^2**

Find average value of y from x = 1 to x = 3



Sol.

$$\langle y \rangle = \frac{\int_1^3 y\,dx}{\int_1^3 dx} = \frac{\int_1^3 x^2\,dx}{\int_1^3 dx} = \frac{\left[\frac{x^3}{3}\right]_1^3}{[x]_1^3} = \frac{\frac{1}{3}[27-1]}{[3-1]} = \frac{26}{6} \Rightarrow \langle y \rangle = \frac{13}{2}$$

Q. **For y = 2 sinθ**

Find average of y from θ = 0 to q = π.

Sol.

$$\langle y \rangle = \frac{\int_0^\pi y\,d\theta}{\int_0^\pi d\theta} = \frac{\int_0^\pi 2\sin\theta\,d\theta}{\int_0^\pi d\theta} = \frac{2[-\cos\theta]_0^\pi}{[\theta]_0^\pi} = \frac{2(2)}{[\pi-0]} = \frac{4}{\pi}$$

$$\langle y \rangle = \frac{4}{\pi}$$

Vectors

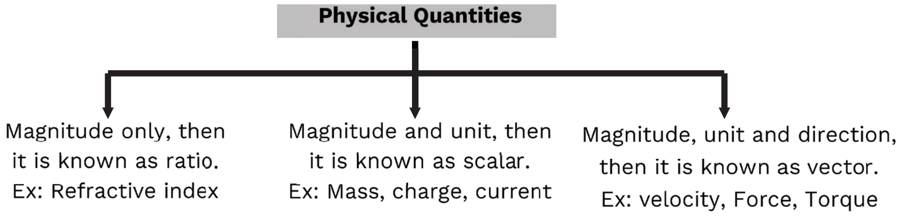
Physical Quantities :

Definition : The quantities by means of which, we described the laws of physics are called physical quantities.

Example : Length, Mass, Time, Force

→ Physical quantities can be measured.

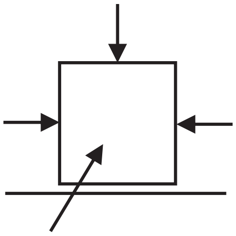
→ Emotions, Feelings, Thinking, Pain are not physical quantities, because they cannot be measured.



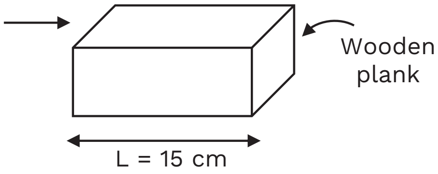
→ If someone asks about your mass and your answer is 60 kg, then person is satisfied because he got the complete answer.

→ If someone asks about location (position) of your school and you only tell the distance of school from your current location, then person will not be satisfied and he will ask about the direction also. Hence position is a vector quantity.

→ A block is kept on a platform and someone asks you to apply a force of 10N on the block. You will ask the person that in which direction you have to apply the force.



∴ Force is a vector quantity because to completely define a force, information of magnitude and direction, both are required.



Length of wooden plank is 15 cm and direction is not required to specify the length. Hence length is a scalar quantity.

- A Physical quantity is vector, if

  → It has magnitude

  → It has direction

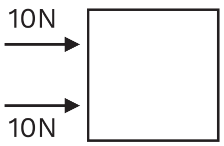
  → It follows rules of vector addition.

- Current is a scalar quantity.

  → Current has magnitude as well as direction but it does not follow laws of vector addition.

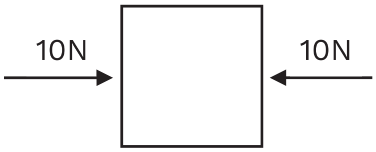
- Vector changes on changing its direction. Value of addition of vectors also changes on changing direction.

  For example : We apply two forces, each of magnitude 10 N in same direction as shown in figure.



Result : Block will move

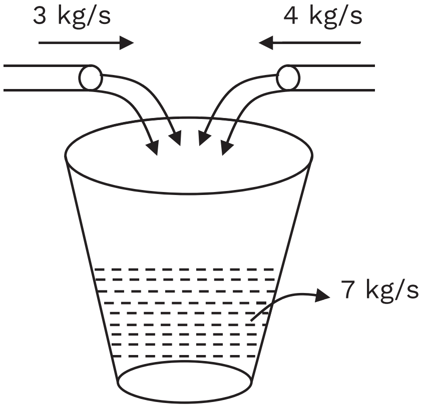
Now, we apply two forces, each of magnitude 10N but in opposite directions as shown in figure.



Result : Block will not move

→ Result changes on changing the direction of vector.
∴ Force is a vector quantity.

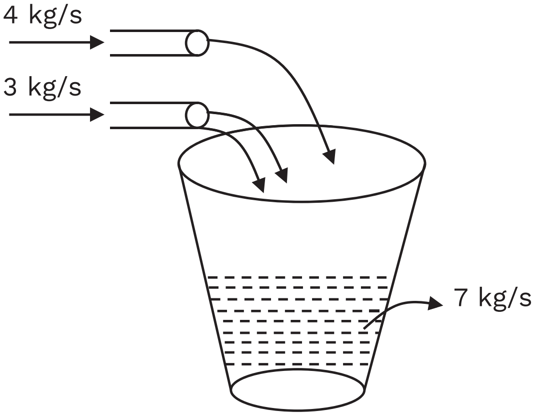
- Water is being filled in a bucket with the help of two pipes.
3 kg/s & 4 kg/s are mass flow rates of water in pipes. Bucket is filled at a rate 7 kg/s.
[If it was like 3 lit/s and 4 lit/s then it is volume flow rate of water in pipe. Bucket will be filled at 7 lit/s.]



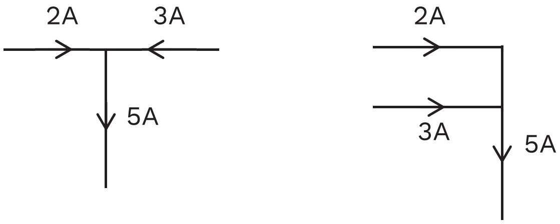
Here also, bucket is filled at 7 kg/s rate.

→ No change in result in above cases on changing the direction of flow.

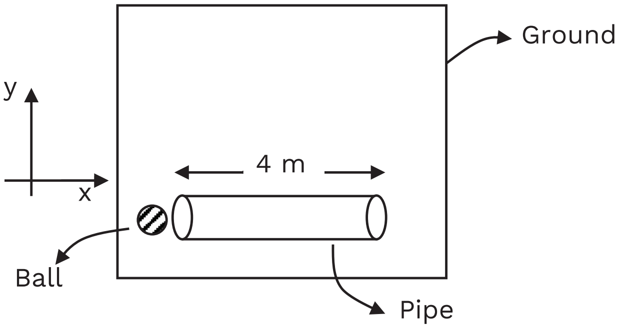
∴ Mass flow rate and volume flow rate are scalar quantities.



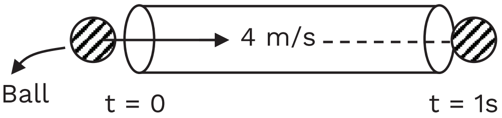
Current :



As there is no change in result, hence current is a scalar quantity.
Now, let's take a ball and pipe setup as shown below.



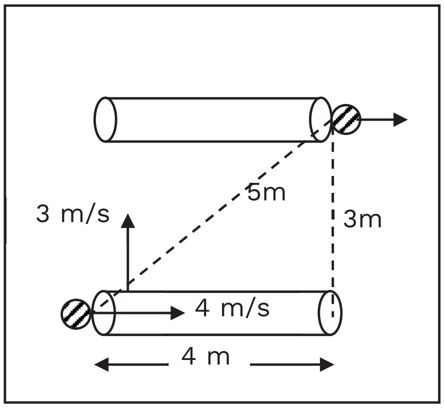
Case I : Ball is given a speed of 4 m/s in x-direction.



Ball will come out of pipe after 1s.

$$t = \frac{4}{4} = 1 \text{ sec}$$

Case II : Ball is given a speed of 4 m/s in x-direction and simultaneously pipe is given a speed of 3 m/s in y-direction.



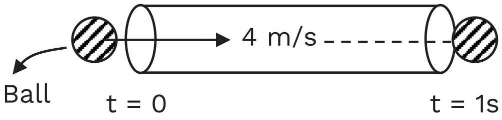
Ball will again come out of pipe after 1s, but this time at a different location.

Ball travels 5m in 1s

∴ Velocity of ball is 5m/s

→ Here ball has been given two velocities, velocity of 4 m/s in x-direction and velocity of 3 m/s in y-direction.

→ Vector addition of two velocities 3 m/s and 4 m/s gives a velocity of 5 m/s in this case.



Case III: Ball is given a velocity of 4 m/s in x-direction and simultaneously pipe is also given a velocity of 3m/s in x-direction.

Ball again comes out of pipe after 1s. This time ball travels 7 m and velocity of ball is 7 m/s.

→ Here resultant of two velocities 3 m/s and 4m/s is 7 m/s.

→ Result changes on changing the direction of velocity.

∴ Velocity is a vector quantity.

VECTOR:

Definition: A physical quantity is said to be a vector is in addition of magnitude and unit, it

→ has specified direction.

→ Obeys the parallelogram law of vector addition.

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

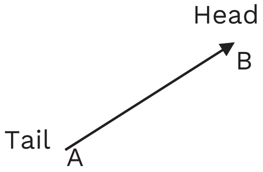
→ And its addition is commutative i.e.

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

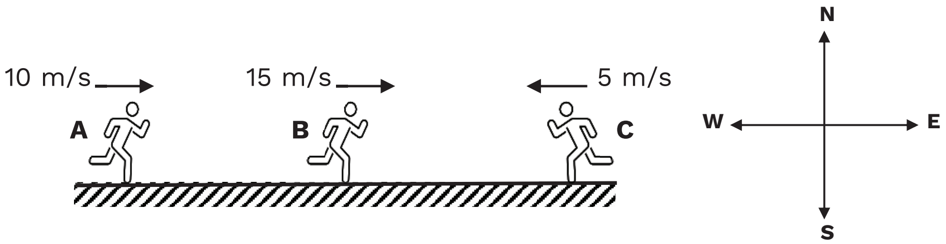
Geometrical representation of vector

Length of arrow : Magnitude of vector quantity.

Direction of arrow head gives its direction.



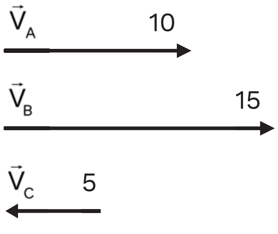
For example, 3 Persons A, B and C are running on road as shown.



Velocity of A, $(\vec{V}_A) = 10\,m/s$, towards East

Velocity of B, $(\vec{V}_B) = 15\,m/s$, towards East

Velocity of C, $(\vec{V}_C) = 5\,m/s$, towards West



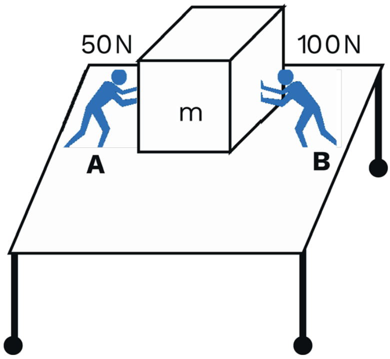
Length of arrow represents the magnitude of vector. Direction of arrow represents the direction of vector.

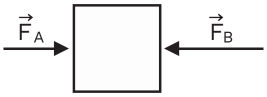
For example :

Two persons A and B are applying forces of 50 N and 100 N respectively on the block as shown.

$\vec{F}_A$ = Force applied by A on block.

= Force applied by B on block.

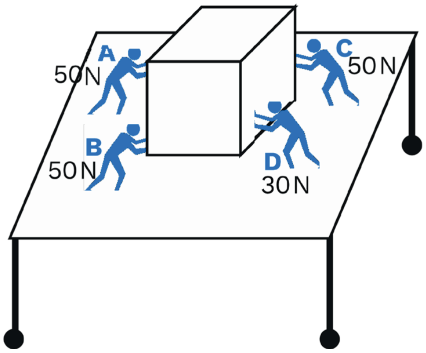


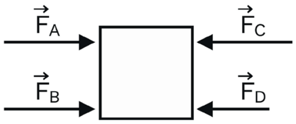


Types of vector :

→ Zero vector
→ Parallel vector
→ Antiparallel vector
→ Equal vector
→ Opposite vector
→ Unit vector
→ Co-linear vector
→ Co-planar vector

Let’s see these various types in the example below:
Four persons A, B, C and D are applying forces on a block as shown in diagram.





Equal vectors: Vectors having same magnitude as well as same direction.

→ $\vec{F}_A$ & $\vec{F}_B$ are equal vectors.

Opposite vectors : Vectors having same magnitude but opposite direction.

→ $\vec{F}_A$ & $\vec{F}_C$ are opposite vectors.

→ $\vec{F}_B$ & $\vec{F}_C$ are opposite vectors.

Parallel vectors : Vector having same direction but different magnitudes.

→ $\vec{F}_C$ & $\vec{F}_D$ are parallel.

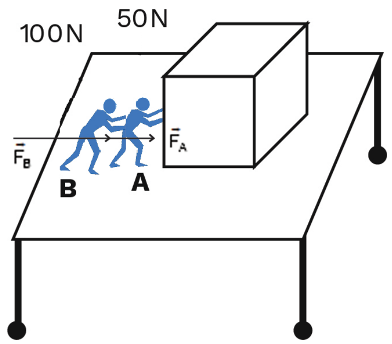
Antiparallel vectors : Vectors having different magnitude but they are in opposite directions.

→ $\vec{F}_A$ & $\vec{F}_D$ are antiparallel.

→ $\vec{F}_B$ & $\vec{F}_D$ are antiparallel.

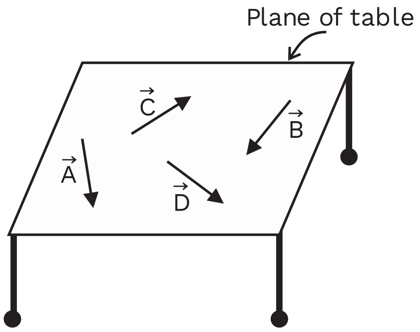
Collinear vectors : Vectors along same line.

→ $\vec{F}_A$ & $\vec{F}_B$ are collinear vectors.



Co-planar vectors: Vectors on same plane.

→ $\vec{A}, \vec{B}, \vec{C}, \& \vec{D}$ are co-planar.



Unit Vector :

→ A vector with magnitude of unity is called unit vector.

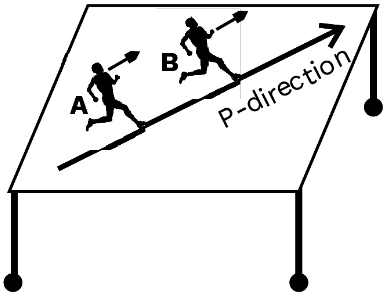
→ Unit vector in direction of $\vec{a}$ is, $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

Let’s have an example to undersatand unit vector.

Two persons A & B are running along direction ‘P’ with velocities 10 m/s and 6 m/s respectively.

→ $V_A = 10\,m/s$ along P-direction

→ $V_B = 6\,m/s$ along p-direction.

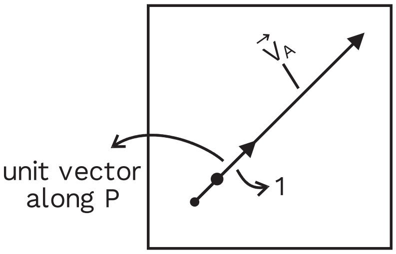


Viewing from the Top

$$\hat{V}_P = \frac{\vec{V}_A}{|\vec{V}_A|} = \text{unit vector in p-direction}$$

$$\vec{V}_B = 6.\hat{V}_P = 6.\frac{\vec{V}_A}{|\vec{V}_A|} = \frac{6}{10}\vec{V}_A$$

$$\vec{V}_B = 0.6\vec{V}_A$$



→ If you want to find a unit vector in any direction then

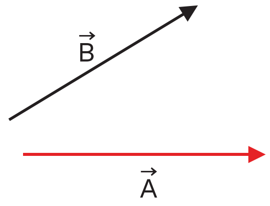
(a) take a vector in that direction.

(b) Divide the vector by its magnitude.

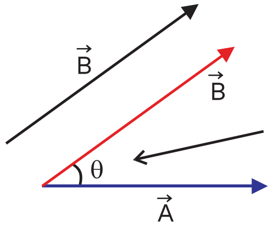
(c) We are left with only direction.

Addition of Vectors :

- Angle between two vectors



→ We can shift a vector parallel to itself.
→ To find the angle between two vectors we have to place them tail to tail.



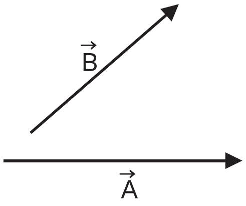
Angle between vectors $\vec{A}$ and $\vec{B}$

Laws of vector addition :

1) Triangle law
2) Parallelogram law

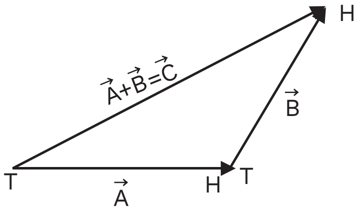
Triangle law :

Given that angle between $\vec{A}$ and $\vec{B}$ is ‘θ’. Find $\vec{A} + \vec{B} = \vec{C}$?

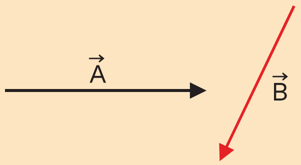


$$\underset{1^{st}}{\vec{A}} + \underset{2^{nd}}{\vec{B}} = \vec{C}$$

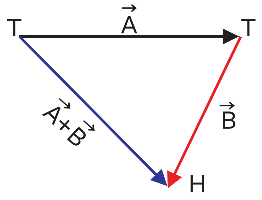
→ Draw the 1st vector as it is.
→ Shift the 2nd vector as it is and keep the tail of 2nd vector on the head of 1st vector.
→ Join the tail of 1st vector with head of 2nd vector and it will give '$\vec{A} + \vec{B}$'



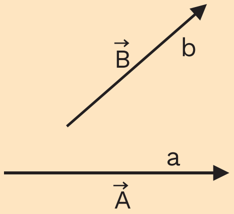
Q. **Find $\vec{A}+\vec{B}$**



Sol.



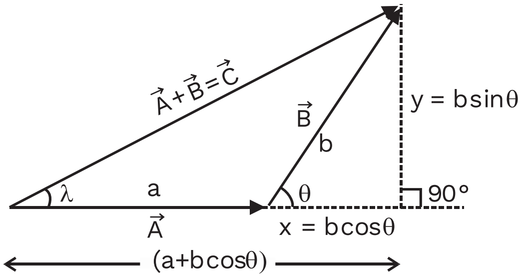
Q. **Angle between $\vec{A}$ & $\vec{B}$ is 'θ' and $|\vec{A}| = a$, $|\vec{B}| = b$. Find $\vec{A}+\vec{B}$?**



Sol. $\frac{x}{b} = \cos\theta \Rightarrow x = b\cos\theta$

$\frac{y}{b} = \sin\theta \Rightarrow y = b\sin\theta$

$(a + b\cos\theta)^2 + (b\sin\theta)^2 = c^2$

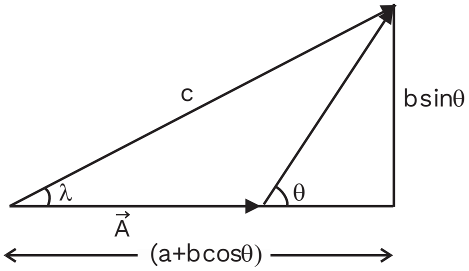


$$\Rightarrow \ a^2 + b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta = c^2$$

$$\Rightarrow \ a^2 + b^2(\sin^2\theta + \cos^2\theta) + 2ab\cos\theta = c^2$$

$$\Rightarrow \ c^2 = a^2 + b^2 + 2ab\cos\theta$$

$$\boxed{c = \sqrt{a^2 + b^2 + 2ab\cos\theta}}$$



Angle of $\vec{B}$ with $\vec{A}$ is 'θ'

Angle of $\vec{C}$ with $\vec{A}$ is 'λ'

$$\boxed{\tan\lambda = \frac{b\sin\theta}{a + b\cos\theta}}$$

Q. **Angle between two vectors $\vec{A}$ and $\vec{B}$ is 60°. $|\vec{A}| = 5, |\vec{B}| = 10$. Then find**

Sol. $|\vec{A} + \vec{B}|$

$|\vec{A}| = a = 5$, $|\vec{B}| = b = 10$, $\theta = 60°$

$$c = \sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{5^2 + 10^2 + 2\cdot5\cdot10\cdot\cos 60°}$$

$$c = \sqrt{25 + 100 + 100 \times \frac{1}{2}} = \sqrt{175}$$

$$|\vec{A} + \vec{B}| = c = \sqrt{175}$$

Angle between $\vec{A}$ & $\vec{C}$

$$\tan\lambda = \frac{b\sin\theta}{a + b\cos\theta} = \frac{10 \times \frac{\sqrt{3}}{2}}{5 + 10 \times \frac{1}{2}} = \frac{10\frac{\sqrt{3}}{2}}{5+5} = \frac{\sqrt{3}}{2}$$

$$\tan\lambda = \frac{\sqrt{3}}{2}$$

$$\lambda = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Q. $|\vec{A}| = 4, |\vec{B}| = 3$ and $|\vec{A}+\vec{B}| = 5$. Find angle between $\vec{A}$ and $\vec{B}$.

Sol. $|\vec{A}+\vec{B}| = c = \sqrt{a^2+b^2+2ab\cos\theta}$

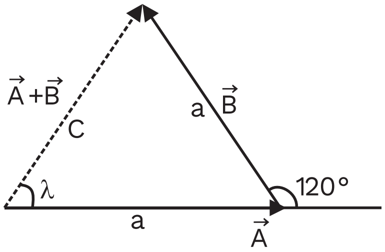
$c^2 = 5^2 = 4^2 + 3^2 + 2\times4\times3\times \cos\theta$

$\Rightarrow \quad 25 + 24\cos\theta = 25 \Rightarrow \quad 24\cos\theta = 25-25 = 0$

$\Rightarrow \quad \cos\theta = 0 \Rightarrow \boxed{\theta = 90^\circ}$

Q. Angle between $\vec{A}$ and $\vec{B}$ is 120°, $|\vec{A}| = a$ and $|\vec{B}| = a$. Find $|\vec{A}+\vec{B}|$ and angle between $\vec{C}$ & $\vec{A}$.

Sol.



$$c = \sqrt{a^2+b^2+2ab\cos\theta}$$

$$c = \sqrt{a^2+a^2+2a\cdot a\cos 120^\circ}$$

$$\cos 120^\circ = -\frac{1}{2}$$

$$c = \sqrt{a^2+a^2+2\cdot a\cdot a\times\left(-\frac{1}{2}\right)}$$

$$c = \sqrt{a^2+a^2-a^2} \quad \Rightarrow c = a$$

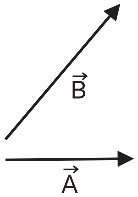
$$\tan\lambda = \frac{a\sin 120^\circ}{a+a\cos 120^\circ} \Rightarrow \tan\lambda = \sqrt{3} \Rightarrow \quad \lambda = 60^\circ$$

Parallelogram law :

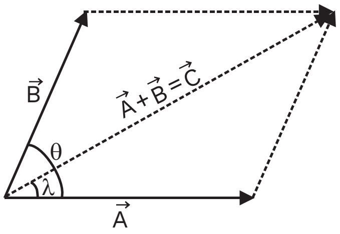
Angle between $\vec{A}$ and $\vec{B}$ is ‘θ’.

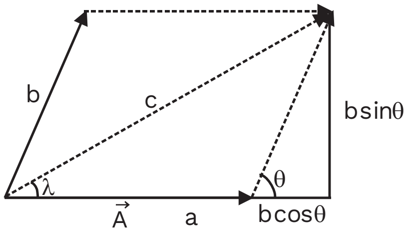
Find, $\vec{A}+\vec{B} = \vec{C}$

As per parallelogram law,



Draw the two vectors such that their tails coincide. Draw a line parallel to $\vec{A}$ passing through head of $\vec{B}$. Draw a line parallel to $\vec{B}$ passing through head of $\vec{A}$. Point of intersection of these lines when joined from the common tail point of $\vec{A}$ and $\vec{B}$, it gives $\vec{A}+\vec{B}$.



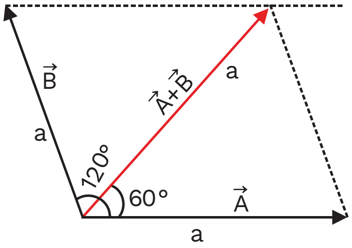


$$c=\sqrt{a^2+b^2+2ab\cos\theta}\ \ \&\ \ \tan\lambda=\frac{b\sin\theta}{a+b\cos\theta}$$

Q. **$|\vec{A}|$ = a, $|\vec{B}|$ = a, Angle between $\vec{A}$ & $\vec{B}$, θ = 120° . Find $\vec{A}+\vec{B}=\vec{C}$**

Sol. $|\vec{C}|=a$ ...... (diagonal of the parallelogram)

Angle of $\vec{C}$ with $\vec{A}$ is 60°.

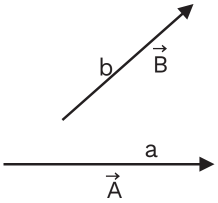


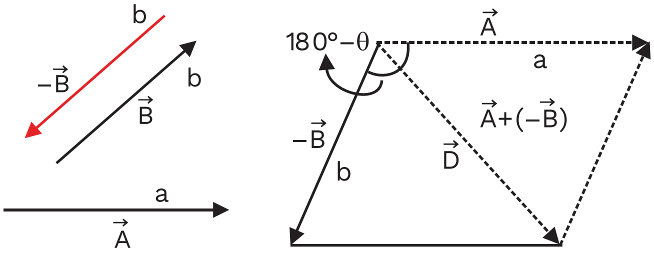
Subtraction of vectors: Subtraction is also a type of addition.

$\vec{A}$ **,** $\vec{B}$ are given. Angle between $\vec{A}$ & $\vec{B}$ is ‘θ’ .

Find $\vec{A}-\vec{B}=\vec{D}$

$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$



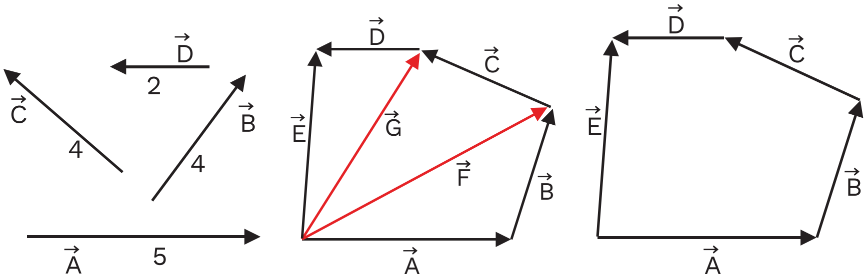


$$|\vec{D}|=\sqrt{a^2+b^2+2ab\cos(180^\circ-\theta)}$$

$$\boxed{d=\sqrt{a^2+b^2-2ab\cos\theta}}$$

Polygon law :

$\vec{A}+\vec{B}+\vec{C}+\vec{D}=\vec{E}$



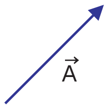
$\vec{A}+\vec{B}=\vec{F}$

$\Rightarrow\quad \vec{F}+\vec{C}+\vec{D}=\vec{E}$

$\vec{F}+\vec{C}+\vec{G}$

$\Rightarrow\quad \vec{G}+\vec{D}=\vec{E}$

Multiplication of a vector by a scalar (number) :



$\vec{B} = 2\vec{A}$

In general

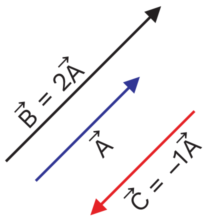
$\vec{B} = n\vec{A}$

If ‘n’ is positive

- magnitude becomes ‘n’ times
- direction remains same

If ‘n’ is negative

- magnitude becomes |n| times
- direction becomes opposite



$\vec{C} = -1\vec{A}$

Components : Resolution of a Vector :

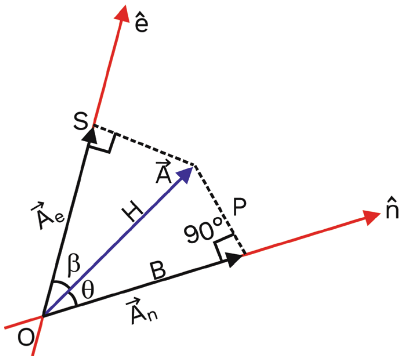
$$\frac{B}{H} = \frac{A_n}{A} \quad \Rightarrow \quad A_n = A \cdot \left(\frac{B}{H}\right)$$

$$\Rightarrow \boxed{A_n = A\cos\theta}$$

'$\vec{A}_n$' is component of '$\vec{A}$' in '$\hat{n}$' direction.

$OS = A_e = A\cos\beta$

'$\vec{A}_e$' is component of '$\vec{A}$' in $\hat{e}$ direction



- Meaning of component is effect.

For example,

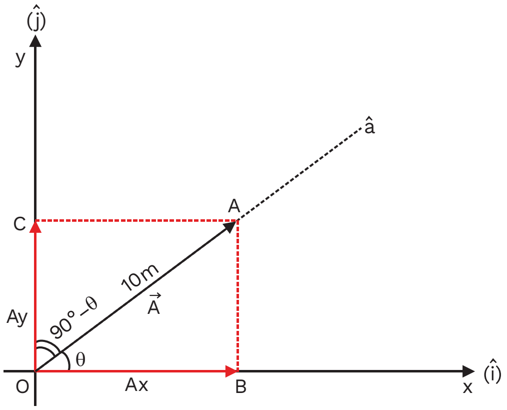
A person moves by 10 m in 'â' direction as shown.

$A_x = A\cos\theta$

$\vec{A}_x = A\cos\theta\hat{i}$

$OC = A_y = A\cos(90° - \theta) = A\sin\theta$

$\vec{A}_y = A\sin\theta\hat{j}$

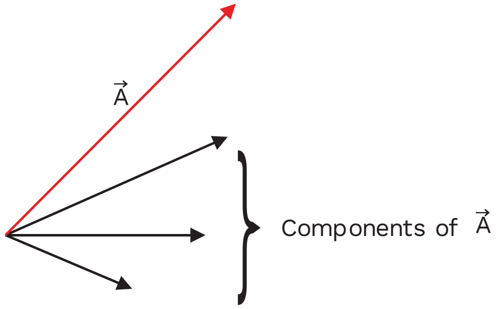


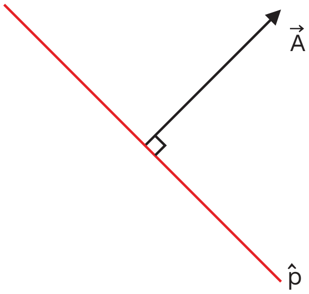
- A vector can have many components.

Component of a vector at 90° is “zero”.

$$A_{90^\circ} = A\cos 90^\circ = 0$$

If a person moves in y-direction then its x co-ordinate will not change.



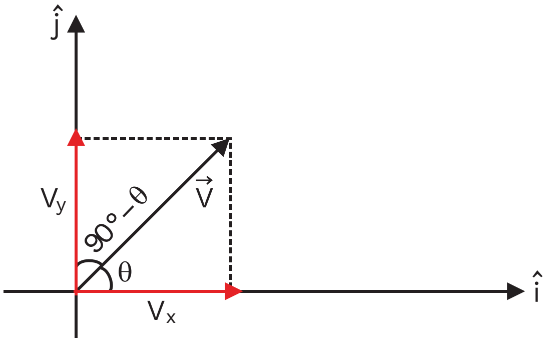


$\theta = 90^\circ$

$\vec{A}_p = 0$

Rectangular Components :

Any set of two components at 90° with each other are called rectangular components.



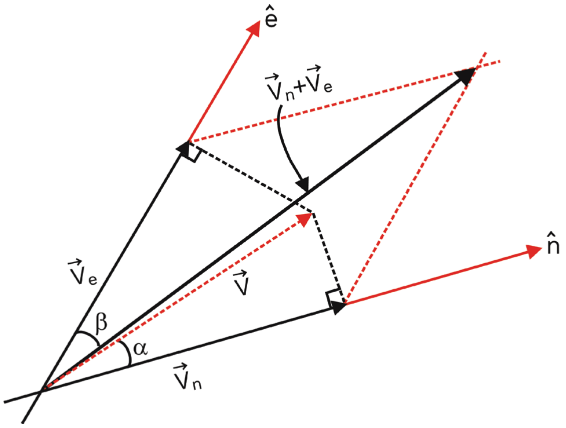
$$V_x = V\cos\theta$$
$$V_y = V\cos(90^\circ - \theta) = V\sin\theta$$
$$\Rightarrow \vec{V}_x = V\cos\theta\,\hat{i}$$
$$\vec{V}_y = V\sin\theta\,\hat{j}$$

- When rectangular components are added, we get original vector.

$$\vec{V} = \vec{V}_x + \vec{V}_y$$

$$\vec{V} = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$$

- For $\alpha + \beta \neq 90°$ $\quad \Rightarrow \vec{V}_e + \vec{V}_n \neq \vec{V}$



→ “A vector can have many sets of rectangular components.”

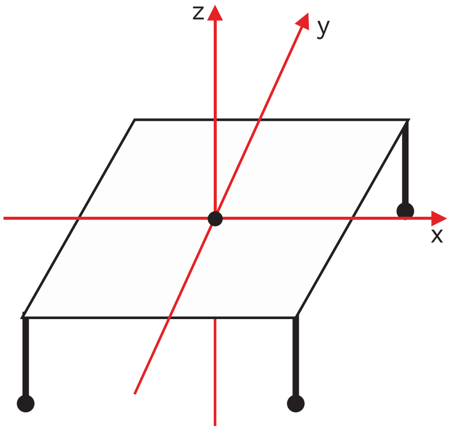
Coordinate System :

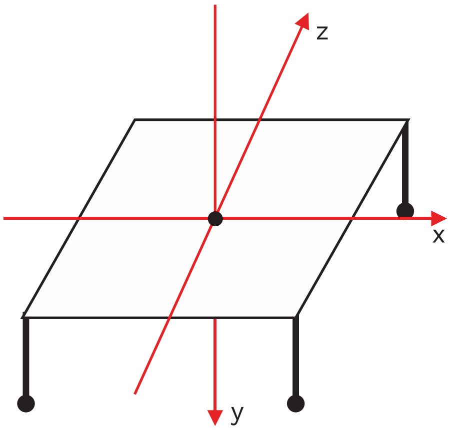
Right handed coordinate system : x, y, z axes are selected by “Right Hand Rule”.

— Put your hand along x-axis

— Curl your four fingers towards y-axis

— Thumb gives direction of z-axis





3-Rectangular Components :

$$V_x = V\cos\alpha$$

$$V_y = V\cos\beta$$

$$V_z = V\cos\gamma$$

$$OS = \sqrt{V_x^2 + V_z^2}$$

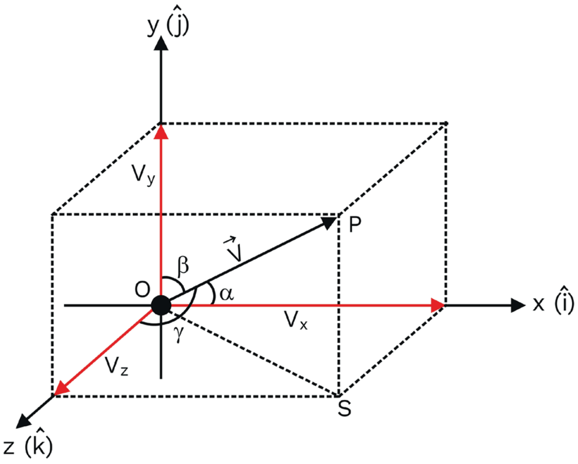
$$OP = \sqrt{OS^2 + SP^2}$$

$$SP = V_y$$

$$OP = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$$

$$\vec{V} = V\cos\alpha\hat{i} + V\cos\beta\hat{j} + V\cos\gamma\hat{k}$$



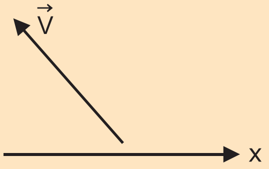
Now,

$$V = \sqrt{(V\cos\alpha)^2 + (V\cos\beta)^2 + (V\cos\gamma)^2}$$

$$\Rightarrow V^2 = V^2[\cos^2\alpha + \cos^2\beta + \cos^2\gamma]$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

Q. **$\vec{V}$ makes an angle 135° with x-axis write $\vec{V}$ in terms of $\hat{i}$ and $\hat{j}$**

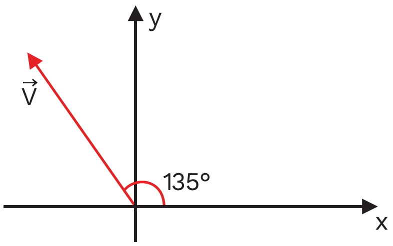


Sol. $\vec{V} = V\cos\theta\hat{i} + V\sin\theta\hat{j}$, Here, $\theta = 135°$

$$\vec{V} = V\cos(135°) + V\sin(135°)$$

$$= V\left(-\frac{1}{\sqrt{2}}\right)\hat{i} + V\left(\frac{1}{\sqrt{2}}\right)\hat{j}$$

$$\vec{V} = -\frac{V}{\sqrt{2}}\hat{i} + \frac{V}{\sqrt{2}}\hat{j}$$



Q. $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$, draw $\vec{A}$. Also find magnitude of $\vec{A}$ i.e. $|\vec{A}|$

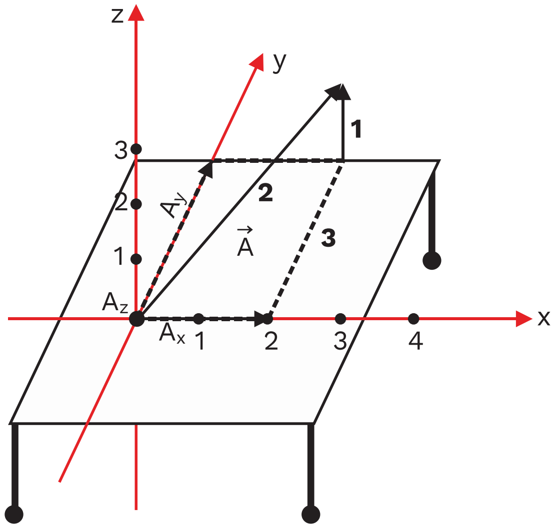
Sol. $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$A_x = 2,\ A_y = 3,\ A_z = 1$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{2^2 + 3^2 + 1^2}$

$|\vec{A}| = \sqrt{14}$



Product of Vectors :

1) Vector Product (Cross Product) ⇒ Result is a vector.

2) Scalar Product (Dot Product) ⇒ Result is a scalar.

- Vector Product of $\vec{A}$ and $\vec{B}$ ⇒ $\vec{A} \times \vec{B} = \vec{C}$ (vector)
- Scalar Product of $\vec{A}$ and $\vec{B}$ ⇒ $\vec{A} \cdot \vec{B} = D$ (scalar)

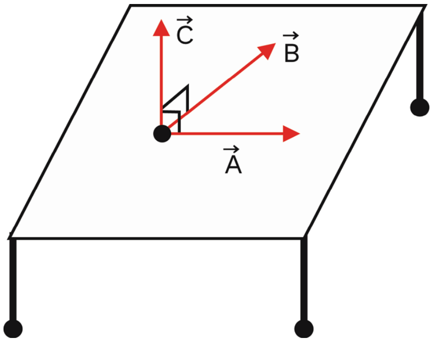
$\vec{A} \times \vec{B} = (ab\sin\theta)\hat{e}$

$\vec{A} \cdot \vec{B} = ab\cos\theta$

Where $|\vec{A}| = a$ and $|\vec{B}| = b$

and ‘θ’ is angle between $\vec{A}$ and $\vec{B}$

Cross Product :



$\vec{A} \times \vec{B} = \vec{C}$

- $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$. both
- $\vec{C}$ is perpendicular to plane formed by $\vec{A}$ and $\vec{B}$.
- Direction of $\vec{C}$ is given by right hand thumb rule.

$\vec{A} \times \vec{B} = \vec{C}$

To find direction of $\vec{C}$

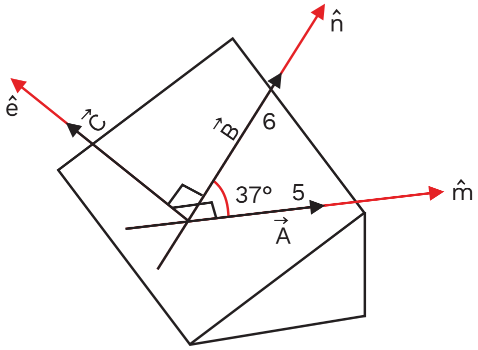
Place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along $\vec{A}$ and when the fingers are closed, they go towards $\vec{B}$. Then, direction of thumb gives the direction of $\vec{C}$.

OR $\vec{A} \times \vec{B} = \vec{C} = AB\sin\theta\,\hat{n}$

- Direction of $\hat{n}$ is found out by same method.

Q. If $\vec{A} = 5\hat{m}$ and $\vec{B} = 6\hat{n}$. Angle between $\hat{m}$ and $\hat{n}$ is 37°. Find $\vec{A} \times \vec{B} = \vec{C}$.

Sol.



$$\vec{A} \times \vec{B} = \vec{C} = AB\sin\theta\,\hat{e}$$
$$= 5 \times 6 \times \sin 37°\hat{e}$$
$$= 5 \times 6 \times \frac{3}{5}\hat{e}$$
$$\vec{C} = 18\,\hat{e}$$

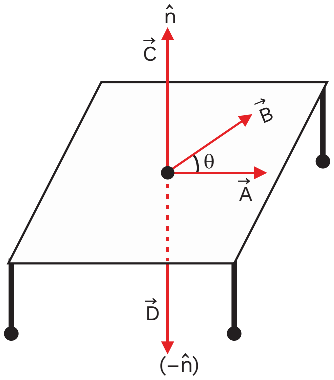
$$\vec{A} \times \vec{B} = \vec{C} = ab\sin\theta\,\hat{n}$$

$$\vec{B} \times \vec{A} = \vec{D} = ba\sin\theta\,(-\hat{n})$$

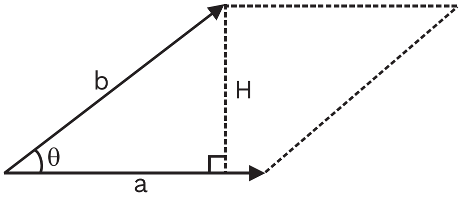
$\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ have same magnitude, but opposite direction.

$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}| = ab\sin\theta$, but

$$\boxed{(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})}$$



$|\vec{A} \times \vec{B}| = ab\sin\theta$ = Base × Height = Area of Parallelogram



Dot Product :

$$\vec{A}\cdot\vec{B} = ab\cos\theta$$

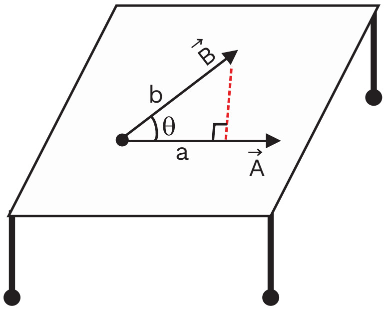
$$\vec{A}\cdot\vec{B} = a(b\cos\theta)$$

= a × length of projection of $\vec{B}$ on $\vec{A}$

= a × component of $\vec{B}$ along $\vec{A}$

$$\vec{A}\cdot\vec{B} = (a\cos\theta)b$$

= length of projection of $\vec{A}$ on $\vec{B}\times b$



Q. $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Find : 1) $\vec{A}\times\vec{B} = \vec{C}$ 2) $\vec{A}\times\vec{B} = d$

Sol.

1) $$|\vec{A}\times\vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

If $\vec{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{B} = 2\hat{i} + 4\hat{j} + 1\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -2 \\ 2 & 4 & 1 \end{vmatrix}$$

$$= [3\times1 - 4\times(-2)]\hat{i} - [2\times1 - 2\times(-2)]\hat{j} + [2\times4 - 2\times3]\hat{k}$$

$$= 11\hat{i} - 6\hat{j} + 2\hat{k}$$

2) $\vec{A} \cdot \vec{B} = d = a_1b_1 + a_2b_2 + a_3b_3$

If $\vec{A} = 2\hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{B} = 2\hat{i} + 4\hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = 2 \times 2 + 3 \times 4 + (-2) \times 1 = 4 + 12 - 2 = 14$

- Angle between $\vec{A}$ and $\vec{B}$

$$\vec{A} \cdot \vec{B} = ab\cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{ab}$$

$a = |\vec{A}|$ and $b = |\vec{B}|$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$, where ‘$\theta$’ is angle between $\vec{A}$ and $\vec{B}$

$$|\vec{A}| = \sqrt{2^2 + 3^2 + (-2)^2} = \sqrt{17}$$

$$|\vec{B}| = \sqrt{2^2 + 4^2 + 1^2} = \sqrt{21}$$

$$\cos\theta = \frac{14}{\sqrt{17}\ \sqrt{21}}.$$

- When two vectors are perpendicular ($\theta = 90°$), their dot product is zero.

$$\vec{A} \cdot \vec{B} = AB\cos 90° = 0$$

JINDAL ADARSH GRAMYA BHARTI

HR. SEC. SCHOOL, KIRODIMAL

NAGAR

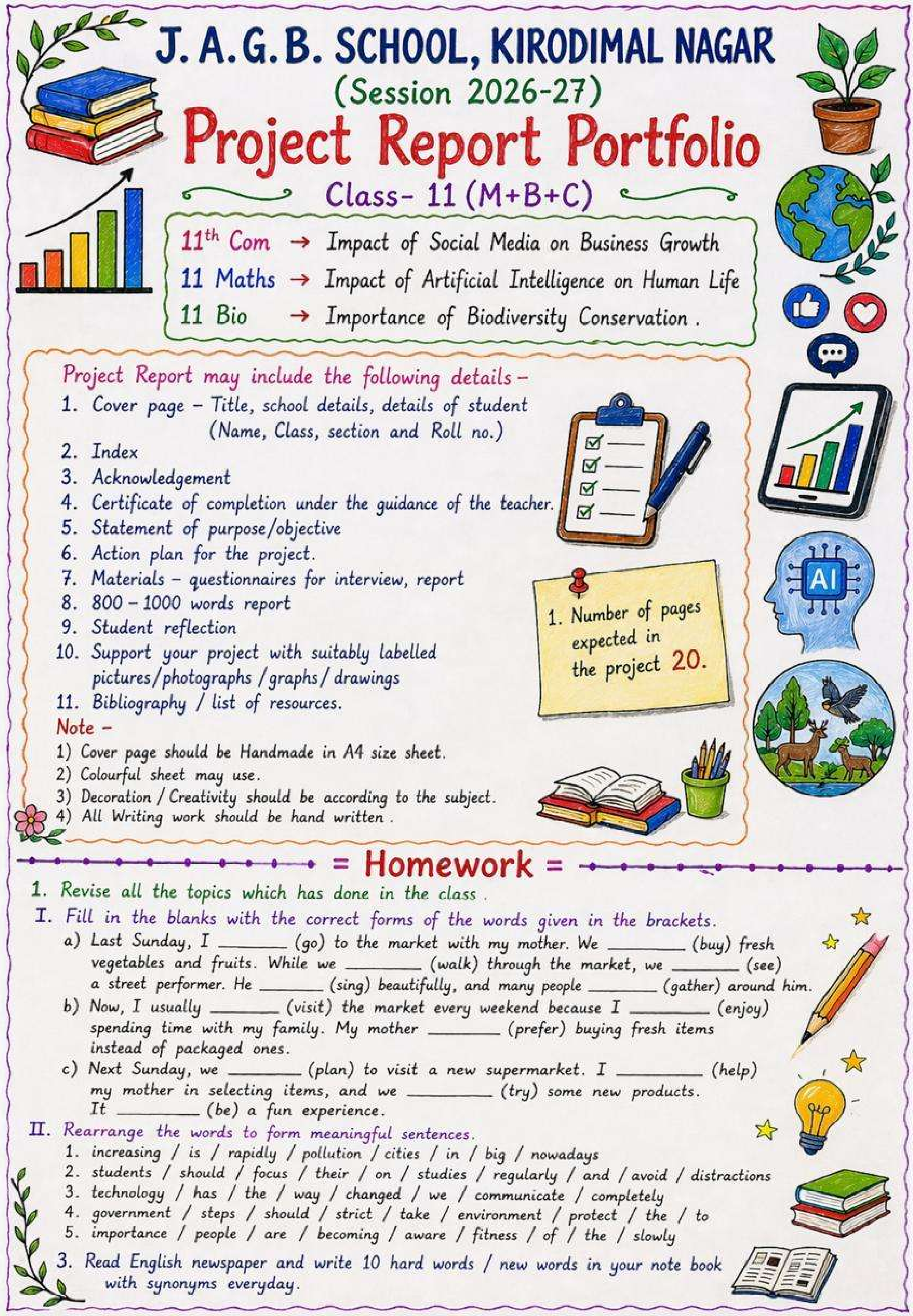
SUMMER VACATION HOMEWORK -

2026

CLASS - XI (COMMERCE)

CBSE

SUBJECT – ENGLISH



J. A. G. B. SCHOOL, KIRODIMAL NAGAR

(Session 2026-27)

Project Report Portfolio

Class- 11 (M+B+C)

11th Com → Impact of Social Media on Business Growth

11 Maths → Impact of Artificial Intelligence on Human Life

11 Bio → Importance of Biodiversity Conservation .

Project Report may include the following details -

1. Cover page - Title, school details, details of student (Name, Class, section and Roll no.)
2. Index
3. Acknowledgement
4. Certificate of completion under the guidance of the teacher.
5. Statement of purpose/objective
6. Action plan for the project.
7. Materials - questionnaires for interview, report
8. 800 - 1000 words report
9. Student reflection
10. Support your project with suitably labelled pictures/photographs/graphs/drawings
11. Bibliography / list of resources.

1. Number of pages expected in the project 20.

Note -

1) Cover page should be Handmade in A4 size sheet.
2) Colourful sheet may use.
3) Decoration / Creativity should be according to the subject.
4) All Writing work should be hand written .

= Homework =

1. Revise all the topics which has done in the class .

I. Fill in the blanks with the correct forms of the words given in the brackets.

a) Last Sunday, I _______ (go) to the market with my mother. We _______ (buy) fresh vegetables and fruits. While we _______ (walk) through the market, we _______ (see) a street performer. He _______ (sing) beautifully, and many people _______ (gather) around him.

b) Now, I usually _______ (visit) the market every weekend because I _______ (enjoy) spending time with my family. My mother _______ (prefer) buying fresh items instead of packaged ones.

c) Next Sunday, we _______ (plan) to visit a new supermarket. I _______ (help) my mother in selecting items, and we _______ (try) some new products. It _______ (be) a fun experience.

II. Rearrange the words to form meaningful sentences.

1. increasing / is / rapidly / pollution / cities / in / big / nowadays
2. students / should / focus / their / on / studies / regularly / and / avoid / distractions
3. technology / has / the / way / changed / we / communicate / completely
4. government / steps / should / strict / take / environment / protect / the / to
5. importance / people / are / becoming / aware / fitness / of / the / slowly

3. Read English newspaper and write 10 hard words / new words in your note book with synonyms everyday.

SUMMER VACATION HOMEWORK

CLASS 11th COMMERCE

SUBJECT – BST

Q.1 Write the difference between Economic and Non economic activities.

Q.2 Explain the concept of Industry and commerce.

Q.3

1. Share
2. Debentures
3. Public Deposit.
4. Capital
5. Assets
6. Fixed Assets
7. Current Assets
8. Retained Earnings.
9. Dividend
10. Credit
11. Debt
12. Barrow
13. Investors.
14. Creditors
15. Shareholders.
16. Liquidity
17. Security.

GENERAL INSTRUCTION

Write all the above questions in your BST copy as SUMMER VACATION HOMEWORK.

Stay Curious, Keep Learning!

PROJECT Work: Insurance

Instructions:

Students are required to prepare a project on the topic “Insurance”. The project should be neatly written on project file paper and must follow the sequence given below:

1. Meaning of Insurance

Explain the concept of insurance and its importance in business and daily life.

2. Principles of Insurance

Explain the main principles of insurance, such as:

Utmost Good Faith
Insurable Interest
Indemnity
Contribution
Subrogation
Proximate Cause

3. Types of Insurance

Describe different types of insurance, for example:

Life Insurance
Fire Insurance
Marine Insurance
Health Insurance

4. Case Study Related to Insurance

Include at least one real or hypothetical case study showing how insurance works in practical situations.

5. Conclusion

Summarize the importance of insurance and what you learned from the project.

CLASS 11th – ACCOUNTANCY

PART A: CONCEPTUAL UNDERSTANDING (SHORT ANSWER)

1. **Define Accounting:**
   Accounting is the process of identifying, measuring, recording, and communicating financial information about a business to various users for decision-making.
2. **The Language of Business:**
   Accounting is often referred to as the **"Language of Business"** because it provides a systematic way to record and communicate financial information, which helps all users understand the financial health and performance of a business.
3. **Book-keeping vs. Accounting:**

Basis	Book-keeping	Accounting
Scope	Narrower in scope. It only involves recording of transactions.	Wider in scope. It includes recording, classifying, summarising, analysing and interpreting of transactions.
Stage	It is the Primary stage of accounting.	It is the Secondary stage of accounting.
Nature of Job	It is a routine and mechanical job.	It is an analytical and interpretative job.

APPLICATION TASK

Classify the following items into Assets, Liabilities, Capital, Revenue, or Expense:

1. Machinery owned by the firm. → **Asset**
2. Salary paid to the office staff. → **Expense**
3. Loan taken from a bank. → **Liability**
4. Cash in hand. → **Asset**
5. Amount withdrawn by the owner for personal use (Drawings). → **Drawings**
6. Commission received. → **Revenue**

PART B: THE VOCABULARY OF FINANCE (BASIC TERMS)

Match the following terms with their correct descriptions:

A. Terms	B. Descriptions
1. Assets	a. Amount invested by the owner in the business.
2. Liabilities	b. Benefits arising from normal business operations.
3. Capital	c. Economic resources owned by the business.
4. Revenue	d. Costs incurred for earning revenue.
5. Expense	e. Amounts owed by the business to outsiders.
6. Drawings	f. Cash or goods withdrawn by the owner for personal use.

PART C: THE ACCOUNTING EQUATION (PRACTICAL APPLICATION)

The fundamental accounting equation is the foundation of the double-entry system:

Assets = Liabilities + Capital

Exercise: Calculate the missing figures in the following scenarios:

① If Total Assets are ₹5,00,000 and External Liabilities are ₹2,00,000, find the Capital.

Capital = Assets – Liabilities = ₹5,00,000 – ₹2,00,000 = **₹3,00,000**

② If the Owner's Equity (Capital) is ₹3,50,000 and Liabilities are ₹1,50,000, find the Total Assets.

Assets = Liabilities + Capital = ₹1,50,000 + ₹3,50,000 = **₹5,00,000**

PART D: ACTIVITY-BASED LEARNING

"The Home Business Project"

1. Imagine you are starting a small "Home Bakery" or a that would be **your Assets.**
   - Oven
   - Mixer
   - Refrigerator
   - Cash in hand
   - Laptop (for orders/marketing)

2. **Identify two possible Liabilities you might incur.**
   - Loan from bank
   - Amount payable to suppliers
3. **List three types of Expenses you would have to pay monthly.**
   - Rent
   - Electricity bill
   - Salaries to helpers

PART E: CRITICAL THINKING

Case Study:

Mr. Sharma started a business by selling his personal car for ₹4,00,000 and investing that entire amount into a new grocery shop. He also borrowed ₹1,00,000 from his brother to buy furniture for the shop.

① What is the total Capital introduced by Mr. Sharma?
Mr. Sharma introduced ₹4,00,000 by selling his personal car.
Total Capital = ₹4,00,000

② What is the value of the Liability in this business?
He borrowed ₹1,00,000 from his brother.
Liability = ₹1,00,000

③ Does the sale of the personal car affect the business books before the money is invested? Why or why not?
No, it does not affect the business books before the money is invested. Because of the **Business Entity Concept**, the business and the owner are separate entities. Only transactions related to the business are recorded in its books.

Tip for Success: Focus on the "Double Entry System" — remember that every transaction has a two-fold effect.

Happy Learning!

ASSINGMENT

1. what is meant by accounting?
2. Give any 4 objectives of accounting?
3. How is profit or loss of a particular period is ascertained?
4. How do we ascertain the financial position of the business?
5. Only financial transactions are recorded in accountancy explain the statement?
6. State any four limitation of accounting?
7. What is the end product of financial accounting?
8. Who are the internal uses of accounting information?
9. Who are the external uses of accounting information?
10. Give the difference between bookkeeping and accounting on the basis of stage special skills and nature of job!
11. Non monetary transactions I am not recorded in the box of accounts explain?
12. Explain the following terms with examples:-

A) capital expenditure

B) non current assets

13. Explain the meaning of any three of the following terms:-

A) liability

B) stock

C) business transactions

D) drawings

14. Give examples of intangible assets?

15. What is voucher?

Project work

1.The Accounting Cycle of a Small Business:

2.Bank Reconciliation Statement

3.Comprehensive Project with GST

4.Project on Depreciation.

(You need to prepare a project file on any one of the topics mentioned above)

Guideline for file

1.Acknowledgement

2.certificate

3.index

4.introduction

5.objective

6.importance

7.limitations

8.conclusion

9.bibleogrpahy.

JINDAL ADARSH GRAMIYA BHARTI
HR. SEC. SCHOOL,
KIRODIMAL NAGAR

VACATION HOMEWORK

CLASS – 11th COMMERCE

SUBJECT – ECONOMICS

Complete your holiday homework neatly in separate files.
Follow the instructions given below:

1 PROJECT WORK

Prepare a project file on **any ONE** of the following topics:

1. Effect on PPC due to various government policies
2. Opportunity Cost as an Economic Tool (Taking real life situations)
3. Effect on Equilibrium Prices in local Market (taking real life situation or recent news)
4. Effect of Price Change on a substitute Goods (taking prices from real life visiting local market)
5. Effect of Price Change on a Complementary Goods (Taking prices from real life visiting local market)
6. Bumper Production – Boon or Bane for the Farmers

Project file should be well-researched, well-organized and creatively presented with relevant charts, data and pictures.

2 WRITTEN WORK

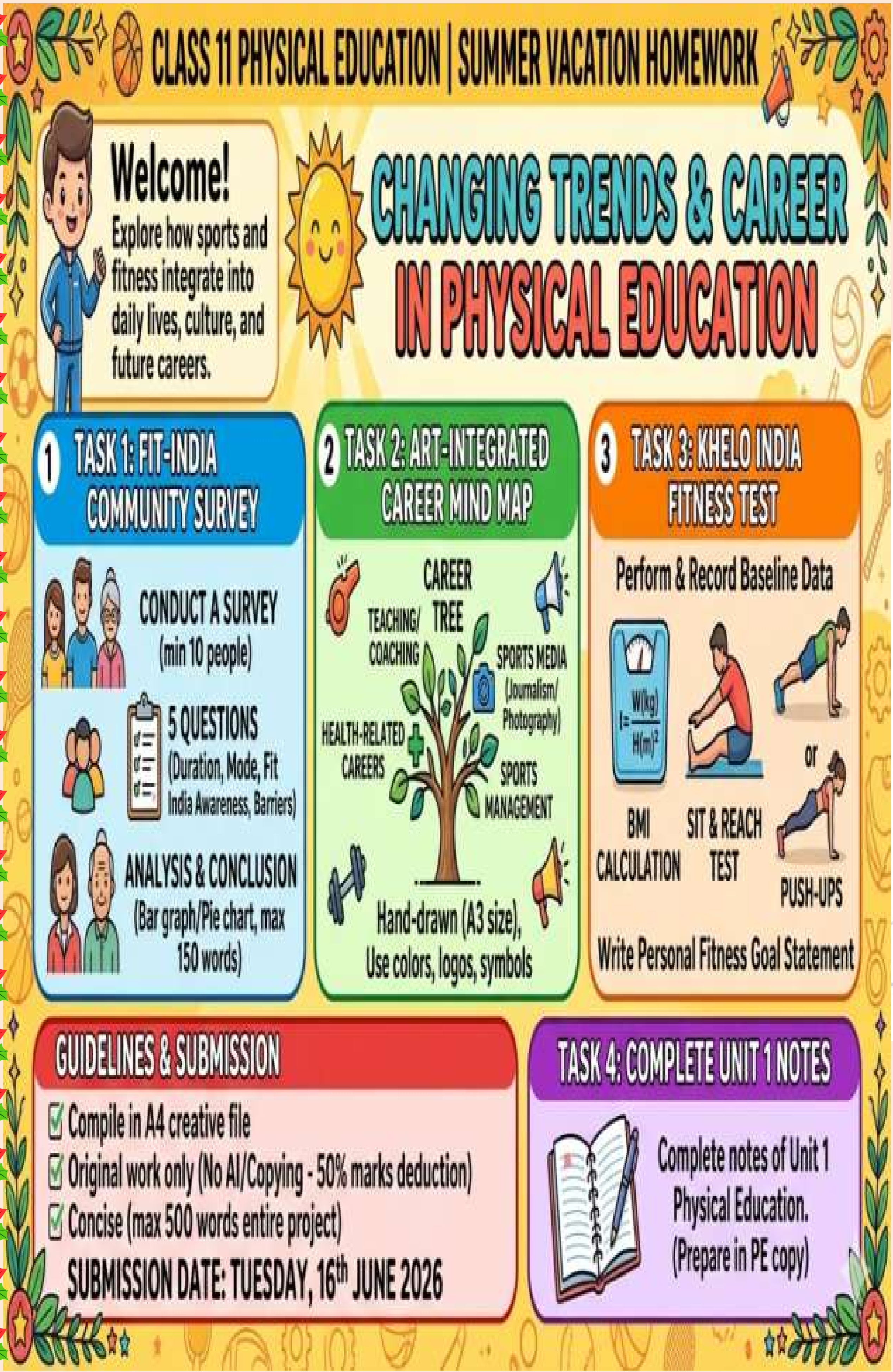
Complete your class notes for Chapter 1 in Statistics.

GENERAL INSTRUCTIONS

- Use separate files for Project Work and Written Work.
- Neatness, presentation and originality of work will be considered.
- Submit your homework on the given date.

PLAN TODAY, WORK SMART, ACHIEVE TOMORROW!

Stay Curious • Stay Inspired • Do Your Best



CLASS 11 PHYSICAL EDUCATION | SUMMER VACATION HOMEWORK

Welcome!

Explore how sports and fitness integrate into daily lives, culture, and future careers.

CHANGING TRENDS & CAREER IN PHYSICAL EDUCATION

1 TASK 1: FIT-INDIA COMMUNITY SURVEY

CONDUCT A SURVEY (min 10 people)

5 QUESTIONS (Duration, Mode, Fit India Awareness, Barriers)

ANALYSIS & CONCLUSION (Bar graph/Pie chart, max 150 words)

2 TASK 2: ART-INTEGRATED CAREER MIND MAP

Hand-drawn (A3 size), Use colors, logos, symbols

3 TASK 3: KHELO INDIA FITNESS TEST

Perform & Record Baseline Data

Write Personal Fitness Goal Statement

GUIDELINES & SUBMISSION

- ☑ Compile in A4 creative file
- ☑ Original work only (No AI/Copying - 50% marks deduction)
- ☑ Concise (max 500 words entire project)

SUBMISSION DATE: TUESDAY, 16th JUNE 2026

TASK 4: COMPLETE UNIT 1 NOTES

Complete notes of Unit 1 Physical Education. (Prepare in PE copy)

★ INFORMATICS PRACTICES ASSIGNMENT ★

CLASS 11th

Write Python programs for the following questions.
Show proper logic and output clearly.

1 Write a program to print "Hello, World!" on the screen.

Logic: Use the print() function to display the text exactly as shown.

2 Write a program to take your name as input and print a greeting message.

Logic: Use input() to take the name and print a personalized greeting.

3 Write a program to add two numbers entered by the user.

Logic: Take two inputs, convert them to numbers, add them and print the result.

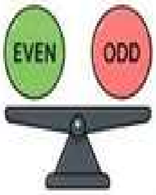
4 Write a program to find the square of a number.

Logic: Take a number as input, square it using * operator and print the result.

n^2

5 Write a program to check whether a number is even or odd.

Logic: Use modulus operator (%) to check and print whether the number is even or odd.



6 Write a program to find the largest of two numbers.

Logic: Take two numbers as input, compare them using if-else and print the largest number.

★ DO YOUR BEST – CODING IS FUN WHEN YOU TRY YOUR BEST! ★

Write Python programs for the following questions. Show proper logic and output clearly.

7 Write a program to find the largest of two numbers.

Logic: Take two numbers as input → use if-else to compare → print the larger number.

8 Write a program to check whether a number is positive, negative, or zero.

Logic: Take a number → use if-elif-else → check >0, <0, or =0 → print result.

9 Write a program to check whether a number is divisible by 5.

Logic: Use modulus operator (%) → if number % 5 == 0 → divisible, else not divisible.

10 Write a program to print your favorite quote on the screen.

Logic: Use print() function to display the quote.

11 Write a program to take your age as input and display it.

Logic: Use input() to take age → print the entered value.

12 Write a program to subtract two numbers entered by the user.

Logic: Take two inputs → convert to numbers → subtract → print result.

13 Write a program to multiply two numbers entered by the user.

Logic: Take two inputs → convert to numbers → multiply them → print the result.

14 Write a program to divide two numbers and display the result.

Logic: Take two inputs → divide first by second → print result (check division by zero).

15 Write a program to find the cube of a number.

Logic: Take a number → multiply it by itself three times (n * n * n) → print result.